# Fall 2005 Math 151

**Exam I Review**

*courtesy: Amy Austin*

(covering sections 1.1-3.3)

## Section 1.1

1. Given $\mathbf{a} = -3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -4\mathbf{i} + 2\mathbf{j}$ , compute and illustrate $-2\mathbf{a} + 3\mathbf{b}$.

2. Given $\mathbf{a}$ and $\mathbf{b}$ above, find a unit vector orthogonal to $\mathbf{a} - \mathbf{b}$.

3. Given $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j}$, find scalars $s$ and $t$ such that $s\mathbf{a} + t\mathbf{b} = 2\mathbf{j}$.

4. An object on the ground is pulled by two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. If $|\mathbf{F_1}|$ is 8 pounds with direction due east and $|\mathbf{F_2}|$ is 20 pounds with direction $N60°E$, find the magnitude of the resultant force.

## Section 1.2

5. Given the points $A(1,4)$, $B(-1,2)$ and $C(3,0)$, find the three angles of $\Delta ABC$.

6. Find the value of $x$ such that the vector from $P(-4,2)$ to $Q(2,1)$ is parallel to the vector from $R(9,x-4)$ to $S(6,2-x)$.

7. Find the vector and scalar projections of $\langle 3,2\rangle$ onto $\langle 1,4\rangle$. Sketch the vector projection.

8. A constant force with vector representation

   $\mathbf{F} = 10i + 18j$ moves an object from the point $(2,3)$ to the point $(4,9)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in Newtons.

9. A woman exerts a horizontal force of 25 pounds as she pushes a crate up a ramp that is 10 feet long and inclined at an angle of 20° above the horizontal. Find the work done.

## Section 1.3

10. Sketch the graph of the vector function

    $\mathbf{r}(t) = 3\cos(2t)\mathbf{i} + 5\sin(2t)\mathbf{j}$.

11. Find parametric equations of the line through the points (2,-3) and (-4,5).

12. Sketch the parametric curve $x = \sqrt{t+2}$, $y = 2t - 1$

13. Given the line $(2+3t)i + (6-4t)j$, find:

    a.) A cartesian equation of the line.

    b.) A vector perpendicular to the line.

## Section 2.2

14. Given $f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 4 & \text{if } x = 1 \\ 4-x & \text{if } x > 1 \end{cases}$

    a.) Sketch the graph of $f(x)$

    b.) Find the following limits:

    $\lim_{x\to -1^-} f(x)$; $\lim_{x\to -1^+} f(x)$; $\lim_{x\to -1} f(x)$;

    $\lim_{x\to 1^-} f(x)$; $\lim_{x\to 1^+} f(x)$; $\lim_{x\to 1} f(x)$

15. Determine the limit: $\lim_{x\to 3} \dfrac{x(4-x)}{(x-3)^2}$

16. Find the vertical and horizontal asymptotes for $f(x) = \dfrac{6x}{3x - x^2}$

## Sections 2.3 and 2.6

17. $\lim_{x\to -1} \dfrac{x^2 - x - 2}{x^2 + 13x + 12}$

18. $\lim_{x\to -2} \left( \dfrac{1}{x+2} + \dfrac{4}{x^2-4} \right)$

19. $\lim_{t\to -\infty} \mathbf{r}(\mathbf{t})$ where $\mathbf{r}(\mathbf{t}) = \left\langle \dfrac{t^2-4}{3t^2-6t+3}, \dfrac{3t}{\sqrt{t^2-4t}} \right\rangle$

20. $\lim_{x\to 0^-} \dfrac{x}{|x^2 - x|}$

21. $\lim_{x\to\infty} (x - \sqrt{x^2+3x+1})$

## Section 2.5

22. Let $f(x) = \begin{cases} x^2 - 2 & \text{if } x < 0 \\ \dfrac{x^2-4}{x-2} & \text{if } 0 \le x < 2 \\ x^3 - 4 & \text{if } x \ge 2 \end{cases}$

    a.) Show $f(x)$ is continuous at $x = 2$ or explain why it is discontinuous.

    b.) Show $f(x)$ is continuous at $x = 0$ or explain why it is discontinuous.

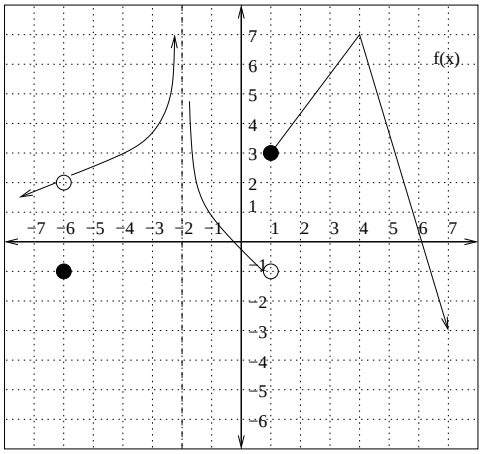
23. State the Intermediate Value Theorem and use it to prove there is a solution to the equation

    $x^3 + 2x + 1 = 0$

24. Given $f(x) = \begin{cases} 3x^3 - 2x + 2a & \text{if } x < 1 \\ 5 & \text{if } x = 1 \\ 3ax - 1 & \text{if } x > 1 \end{cases}$

find the value of $a$ that makes $f(x)$ continuous everywhere, if possible. If it is not possible, be sure to support your answer.

25. Refer to the graph given below to find all $x$ coordinates where $f(x)$ is not continuous. Support your answer.




## Sections 2.7 and 3.1

26. A ball is thrown into the air with a speed of 40 ft/sec. The height of the ball after $t$ seconds is given by $h(t) = 40t - 16t^2$.

a.) Find the average velocity of the ball from $t = 2$ to $t = 2.5$.

b.) Using the limit definition, find the instantaneous velocity of the ball at $t = 2$ seconds.

27. Given $f(x) = \sqrt{x+1}$

a.) Find the slope of the secant line joining the points $(2, \sqrt{3})$ and $(3, 2)$

b.) Using the limit definition, find the slope of the tangent line to the graph of $f(x)$ at the point $(3, 2)$. What is the equation of this tangent line?

28. Compute the derivative of $f(x) = \dfrac{x}{x+1}$, using the definition of the derivative.

29. $f(x) = \begin{cases} ax + 3 & \text{if } x \leq 3 \\ bx^2 - 2x + 8 & \text{if } x > 3 \end{cases}$,

find the values of $a$ and $b$ that make $f(x)$ continuous and differentiable everywhere.

30. Refer to the graph given in number 25 to determine where $f(x)$ is not differentiable.

## Section 3.2

31. Find the derivative of the following functions.

a.) $f(x) = x^3(x^4 + 3x^2 + 2x)$

b.) $g(x) = \dfrac{x^2 + x}{x^3 - 1}$

c.) $f(t) = \sqrt{t^3} - \dfrac{5}{\sqrt{t}} + \pi^2$

d.) $\mathbf{r(t)} = \left\langle 10\sqrt{t}, 30t - \dfrac{16}{t^2} \right\rangle$

32. If $f(x) = |x^2 - 9|$, find where $f(x)$ is not differentiable. Find a formula for $f'(x)$ and sketch the graph of $f(x)$ and $f'(x)$.

33. Find the equation of the tangent line to the graph of $f(x) = x^2 + \dfrac{5}{x}$ where $x = 1$.

34. If $6x + 2y = 3$ is the equation of the tangent line to the graph of $f(x)$ at $x = 1$, what is $f'(1)$?

35. If $f(x) = g(x)\,(h(x) + 5x)$, find $f'(1)$ given the table below.

| $x$ | $g(x)$ | $h(x)$ | $g'(x)$ | $h'(x)$ |
|---|---|---|---|---|
| 1 | 2 | -3 | 4 | -1 |

36. Work problem 61, page 169 in Stewart text book.

37. Draw a diagram to show there are two tangent lines to the parabola $y = x^2$ that pass thru the point $(0, -4)$. Find the $x$ coordinates where these tangent lines touch the parabola.

38. Suppose a curve C is represented by the vector equation

$\mathbf{r(t)} = \langle 2t + 1, t^2 - 2t \rangle$:

a.) Find the tangent vector to the curve at the point $(3, -1)$.

b.) Find parametric equations for the tangent line to the curve at the point $(3, -1)$.

39. Find all points on the graph of $f(x) = x\sqrt{x}$ where the tangent line is parallel to the line $6x - 2y + 3 = 0$.

## Section 3.3

40. A particle moves according to the equation of motion $s = 4t^3 - 9t^2 + 6t + 2$, $t \geq 0$, where $s$ is measured in feet and $t$ in seconds. When is the particle at rest? When is the particle moving in the negative direction?