MATH 151
FALL 2009

SAMPLE EXAM II

Part I - Multiple Choice

1. Use differentials to approximate $\sqrt{11}$.

a) $\frac{8}{3}$ b) $\frac{11}{3}$ c) $\frac{23}{6}$ d) $\frac{21}{6}$ e) $\frac{10}{3}$

2. If $f(x) = 3x\cos^2(x^2)$, find $f'(0)$.

a) 0 b) -3 c) 3 d) 1 e) -9

3. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t + 4$ where t is measured in seconds and $s(t)$ in meters. Which of the following statements is false?

a) The object is at rest at time $t = 1$ second.

b) The object has positive acceleration at $t = 3$ second

c) The object is moving in the positive direction between $t = 0$ and $t = \frac{1}{2}$

d) The total distance traveled in the first 3 seconds is 30 meters.

e) All of the above statements are true.

4. Two sides of a triangle are fixed at 4cm and 6cm and the angle between them is increasing at a rate of .02 radians per second. How fast is the area of the triangle increasing when the angle between them is $\frac{\pi}{6}$?

a) $(.12)\sqrt{3}$ b) $\frac{.02}{6}$ c) $\frac{.02}{6\sqrt{3}}$ d) .12 e) $12\sin(.02)$

5. Let $f(x) = (1 + x^2)^{\frac{3}{2}}$. Then $f''(0) =$

a) 3 b) 0 c) 6 d) $\frac{3}{4\sqrt{2}}$ e) $\frac{3}{4}$

6. What values of r does $y = e^{rx}$ satisfy the equation $y'' + y' - 2y = 0$?

a) $r = 2$, $r = -1$ b) $r = 2$, $r = 3$ c) $r = \pm\sqrt{2}$

d) $r = -2$, $r = 1$ e) $r = 2$, $r = -2$

7. The function $f(x) = x^3 + 5x - 1$ is one-to-one. Let $g = f^{-1}$. Then $g'(5) =$

a) 8 b) $\frac{1}{80}$ c) $\frac{8}{25}$ d) $\frac{1}{8}$ e) 80

8. Given the curve parametrized by $x = t^3 - 3t^2 - 9t + 1$, $y = t^3 + 3t^2 - 9t + 1$, at which point does the curve have a vertical tangent?

a) $(1, -3)$ b) $(6, 12)$ c) $(-10, 6)$

d) $(-1, 3)$ e) $(1, 1)$

9. $\lim_{x \to 0} \frac{4 \cos x - 4 + 3 \sin x}{5x} =$

a) $\frac{4}{5}$ b) $-\frac{4}{5}$ c) $\frac{3}{5}$ d) 1 e) 0

10. Find the slope of the line tangent to the curve given by $y^2 + xy = 8$ at the point $(-2, -2)$.

a) -2 b) $-\frac{10}{3}$ c) $-\frac{1}{3}$ d) -3 e) 0

11. Which of the following statements is true about the curve $(2 + \cos t)\mathbf{i} + (1 + \sin t)\mathbf{j}$?
a) Clockwise movement around the circle $(x-2)^2 + (y-1)^2 = 1$

b) Counterclockwise movement around the circle $(x-2)^2 + (y-1)^2 = 1$

c) Clockwise movement around the ellipse $x^2/4 + y^2 = 1$

d) Counterclockwise movement around the ellipse $x^2/4 + y^2 = 1$

e) None of the above statements is correct.

12. Let $f(x)$ be a differentiable function and let $g(x) = 3x^2 - 1$. Let $H(x) = f(g(x))$, the composit of f and g. If $f(0) = 1$, $f'(0) = -1$, $f(1) = 3$, $f'(1) = 2$, $f(2) = -1$, $f'(2) = 5$, find $H'(1)$.

a) 30 b) 12 c) -6 d) 6 e) 5

13. $\lim_{x \to \infty} 3^{1-x} =$

a) 0 b) ∞ c) $-\infty$ d) 1 e) 3

14. Find the domain and range of the inverse of $f(x) = \dfrac{3x-5}{7x+2}$

a) Domain: All real numbers except $\dfrac{2}{7}$; Range: All real numbers except $-\dfrac{3}{7}$

b) Domain: All real numbers except $-\dfrac{2}{7}$; Range: All real numbers except $\dfrac{3}{7}$

c) Domain: All real numbers except $\dfrac{3}{7}$; Range: All real numbers except $-\dfrac{2}{7}$

d) Domain: All real numbers except $\dfrac{5}{3}$; Range: All real numbers.

e) None of the above is correct.

15. If $\langle \cos 3t, t \rangle$ is the position of an object at time t, find the acceleration of the object at time $t = \dfrac{\pi}{9}$.

a) $\left\langle \dfrac{1}{2}, 0 \right\rangle$ b) $\left\langle -\dfrac{1}{2}, 0 \right\rangle$ c) $\left\langle -\dfrac{9}{2}, 0 \right\rangle$

d) $\left\langle \dfrac{9}{2}, 0 \right\rangle$ e) $\langle 3, 0 \rangle$

16. If $f(x) = e^{x \tan x}$, find $f'(x)$.

a) $f'(x) = e^{x \tan x}$

b) $f'(x) = \sec^2 x e^{x \tan x}$

c) $f'(x) = (\tan x + x \sec^2 x) e^{x \tan x}$

d) $f'(x) = (\tan x + x \sec x \tan x) e^{x \tan x}$

e) $f'(x) = x \tan x e^{x \tan x - 1}$

17. Find the equation of the tangent line to the graph of $x = e^{2t}$, $y = te^t$ at the point $(1, 0)$.

a) $y = 2x - 1$ b) $y = 4x - 4$ c) $y = \dfrac{1}{2}x - \dfrac{1}{2}$

d) $y = \dfrac{1}{3}x - \dfrac{1}{3}$ e) $y = x - 1$

18. Find the quadratic approximation for $f(x) = \dfrac{1}{x}$ at $x = 1$.

a) $x^2 - 3x + 3$ b) $x^2 - x + 2$ c) $x^2 - 2x + 1$

d) $x^2 + 4x + 5$ e) $x^2 + x - 3$

Part II - Work Out Problems

No calculators

19. a.) Find the linear approximation for $f(x) = \sqrt[4]{x+1}$ at $x = 0$.
    b.) Use part a.) to obtain an approximation to $\sqrt[4]{1.01}$

20. The position of a particle is given by $\mathbf{r}(t) = \left\langle \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right\rangle$. Find the velocity and speed of the particle when $t = 0$.

21. The radius of a sphere was given to be 8 inches with a maximum possible error in measurement of 0.01 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere.

22. Find all values of x between 0 and 2π where the tangent line to $f(x) = 2x - \tan x$ is horizontal.

23. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute, how fast is the water level rising when the height of the water is 1 foot?

24. Find an equation (in any form) of the line tangent to the curve $\mathbf{r}(t) = (t^6 + t^3)\mathbf{i} + (t^4 + t^2)\mathbf{j}$ at the point where $t = 1$.

25. A rope is attached to the bow of a boat coming in for the evening. Assme the rope is drawn in over a pully 5 feet higher than the bow at a rate of 2 feet per second. How fast is the boat docking when the length of the rope from the bow to the pully is 13 feet?