Fall 2005 Math 151

Week in Review II

courtesy: Amy Austin

(covering sections 1.3, 2.2, 2.3)

Section 1.3

1. Find a cartesian equation for the following parametric curves. Sketch the curve and be sure to state the domain and range.

   a.) $x = 1 - t$, $y = 4t + 2$

   b.) $x = t + 2$, $y = 3 - 2t$, $0 \le t \le 3$

   c.) $x = \sqrt{t}$, $y = t - 4$

   d.) $x = 2\cos t$, $y = 3\sin t$, $0 \le t \le \pi$

   e.) $x = 3 + \sin t$, $y = 2 + \cos t$

   f.) $x = \cos t$, $y = \sin^2 t$

2. An object is moving in the xy-plane and the position of the object after t seconds is given by $\mathbf{r}(\mathbf{t}) = \langle t + 4, t^2 + 2t \rangle$.

   a.) Find the position of the object at time $t = 2$.

   b.) At what time does the object reach the point $(7, 15)$?

   c.) Does the object pass through the point $(9, 20)$?

   d.) Find a cartesian equation for the path of the object.

3. Find parametric equations for the line that passes thru the points $(0, 3)$ and $(-3, 5)$.

4. Determine whether the following lines are parallel or perpendicular. If they are not parallel, find the point of intersection.

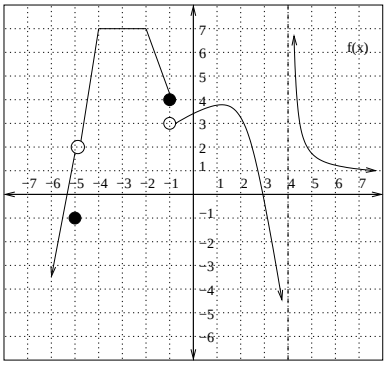
   L1: $\mathbf{r}(\mathbf{t}) = (-4 + 2t)\mathbf{i} + (5 + t)\mathbf{j}$

   L2: $\mathbf{r}(\mathbf{t}) = (2 + 3t)\mathbf{i} + (4 - 6t)\mathbf{j}$

5. Find parametric equations for the line that passes thru the point (1,5) and is perpendicular to the line $x = 1 + t$, $y = 5 + 4t$.

Section 2.2

6. Use the graph of $f(x)$ below to compute the following limits:



   a) $\lim_{x \to -1^-} f(x)$ b) $\lim_{x \to -1^+} f(x)$

   c) $\lim_{x \to -1} f(x)$ d) $\lim_{x \to -5} f(x)$

   e) $\lim_{x \to 4^+} f(x)$ f.) $\lim_{x \to -3} f(x)$

7. $\lim_{x \to 4^+} \dfrac{x - 1}{x - 4}$

8. Find the vertical asymptote(s) of the function $f(x) = \dfrac{x - 7}{x^2 - 6x - 7}$ and determine the behavior of the function near the vertical asymptote(s).

Section 2.3

Compute the exact value of the following limits:

9. $\lim_{x \to 2}(x^2 - x - 3)$

10. $\lim_{x \to -2} \dfrac{x^2 + 5x + 6}{x^2 + x - 2}$

11. $\lim_{x \to 1} \dfrac{\sqrt{x^2 + 2x} - \sqrt{3}}{x - 1}$

12. $\lim_{x \to 3} \left(\dfrac{1}{x} - \dfrac{1}{3}\right)\left(\dfrac{1}{x - 3}\right)$

13. $\lim_{x \to 2} \dfrac{x - 4}{x - 2}$

14. $\lim_{t \to 1} \left(\dfrac{1}{t - 1} - \dfrac{2}{t^2 - 1}\right)$

15. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} \sqrt{x^2 + 16} & \text{if } x \le 3 \\ x^3 - 10 & \text{if } x > 3 \end{cases}$

16. $\lim_{x \to 2} \dfrac{|\, 3x - 6 \,|}{x^2 - 2x}$

17. $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$

18. Sketch the graph of $f(x) = \dfrac{|x-4|}{x-4}$ and give all x coordinates where the limit does not exist.

19. $\lim_{x \to 1} f(x)$ if it is known that $4x \leq f(x) \leq x + 3$ for all x in $[0, 2]$.

20. $\lim_{x \to 0} x^4 \sin(\frac{1}{x})$