Fall 2005 Math 151

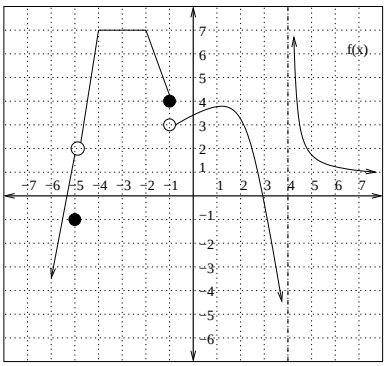
Week in Review 3

courtesy: Amy Austin

(covering sections 2.5 - 2.7)

Section 2.5

1. Referring to the graph, explain why the function $f(x)$ is or is not continuous (you decide which) at $x = -1$, $x = -5$, $x = 4$, and $x = -2$.



2. Where is the function

$$f(x) = \begin{cases} \frac{1}{x} + 2 & \text{if } x < 1 \\ 7 & \text{if } x = 1 \\ 3x & \text{if } x > 1 \end{cases}$$

not continuous? Support your answer.

3. If $f(x) = \dfrac{x+3}{x^2+5x+6}$, find all values of $x = a$ where the function is discontinuous. For each discontinuity, find the limit as x approaches a.

4. If $g(x) = x^5 - 3x^2 + 1$, use the Intermediate Value Theorem to prove there is a solution to the equation $g(x) = -2$.

5. Use the Intermediate Value Theorem to find two consecutive integers a and $a+1$ such that the interval $[a, a+1]$ contains a solution to the equation $x^3 + 2x + 1{=}0$.

6. Find the values of c and d that will make

$$f(x) = \begin{cases} dx - c & \text{if } x \le 0 \\ cx + d & \text{if } 0 < x \le 3 \\ x^2 - dx - 11 & \text{if } x > 3 \end{cases}$$

continuous on all real numbers.

Section 2.6

7. Compute the following limits:

a.) $\lim\limits_{x\to\infty} \dfrac{4x^3 - 6x^4}{2x^4 - 9x + 1}$

b.) $\lim\limits_{t\to\infty} \dfrac{t^9 - 4t^{10}}{t^8 + 2t^2 + 1}$

c.) $\lim\limits_{x\to\infty} \dfrac{\sqrt{2 + 25x^2}}{4 - 3x}$

d.) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{3x^2 + 1}}{4x - 3}$

e.) $\lim\limits_{x\to\infty} (\sqrt{x^2 + 5x + 1} - x)$

f.) $\lim\limits_{x\to-\infty} (x + \sqrt{x^2 + x + 2})$

8. Find all horizontal and vertical asymptotes:

a.) $f(x) = \dfrac{x^3}{x^3 - x}$

b.) $f(x) = \dfrac{3 - x}{\sqrt{x^2 + 1}}$

c.) Refering to the graph in problem 1, find all vertical and horizontal asymptotes.

Section 2.7

9. Using the limit definition for slope, find the equation of the tangent line to the graph of $f(x)$ at the indicated value:

a.) $f(x) = \sqrt{x+1}$ at the point $(3, 2)$

b.) $f(x) = \dfrac{x}{1-x}$ at $x = 0$

10. The displacement (in meters) of a particle moving along a straight path is given by $s(t) = t^2 - 8t + 18$, where t is measured in seconds. Compute:

a.) The average velocity of the particle over the time interval $[3, 4]$.

b.) The instantaneous velocity at time $t = 3$.

11. Given $\mathbf{r(t)} = \langle t^2, 2t - 2 \rangle$:

a.) Find the tangent vector to the curve $\mathbf{r(t)}$ at the point $(4, 2)$.

b.) Find parametric equations for the tangent line to the curve at the point $(4, 2)$.

c.) Eliminate the parameter to find a cartesian equation of the tangent line.