Fall 2005 Math 151

courtesy: Amy Austin
(covering sections 3.7-3.12)

Section 3.7

1. Find the angle between the tangent vector and the position vector for $\mathbf{r(t)} = \langle t^2, 2t^3 \rangle$ at the point where $t = -1$.

2. Find the vector and parametric equations of the line tangent to $\mathbf{r(t)} = \langle t^3 + 2t, 4t - 5 \rangle$ at the point where $t = 2$.

3. Sketch the curve $\mathbf{r(t)} = \langle t^2, t \rangle$. Find the tangent and unit tangent vector to the curve at the point $(4, 2)$. Draw the position and tangent vector along with the sketch of the curve at the point $(4, 2)$.

4. Find the angle of intersection of the curves
$\mathbf{r_1(s)} = \langle s - 2, s^2 \rangle$ and $\mathbf{r_2(t)} = \langle 1 - t, 3 + t^2 \rangle$

Section 3.8

5. Given $y(t) = 4t^3 - 15t^2 + 12t + 5$ is the position of an object at time t. Assume y is measured in feet and t is measured in seconds.

   a) Find the velocity and acceleration of the object at time $t > 0$.

   b) Find the times when the velocity is 0 and find the acceleration of the object at these times.

6. Given $f(x) = \sqrt{3x + 1}$, compute $f'''(0)$

7. Find the 83rd derivative of $f(x) = \sin(2x)$.

8. Suppose the position of a particle at time t is given by $\mathbf{r}(t) = \sin(2t)\mathbf{i} + (\cos 2t)\mathbf{j}$. Find the velocity, speed, and acceleration of the particle when $t = \dfrac{\pi}{2}$. Plot the position, velocity, and acceleration vectors along with the sketch of the curve.

Section 3.9

9. A curve is given parametrically by
$x = t^3 - 3t^2$, $y = t^3 - 3t$.

   a) Find the equation of the line tangent to the curve at the point where $t = -1$

   b) Find all the points on the curve where the tangent line is horizontal or vertical

10. If $x = t^3 - 5t$ and $y = 6t - 1$, find the slope of the tangent line at the point $(-2, 11)$

Section 3.10

11. A plane flying horizontally at an altitude of 1 km passes directly over a radar station at a speed of 500 km/hr. Find the rate at which the distance from the plane to the station is increasing when it is 2 km away from the station.

12. A filter in the shape of a cone is $10cm$ high and has a radius of $20cm$ at the top. A solution is poured in at the rate of $2cm^3/min$. Find the rate at which the height of the residue is increasing when the height is $2cm$.

13. The length of a rectangle is increasing at a rate of 2 feet per second, while the width is decreasing at a rate of 1 foot per second. When the length is 5 feet and the width is 3 feet, how fast is the area changing?

14. The volume V of a cube with sides length x inches is changing with respect to time. At a certain instant of time, the sides of the cube are 5 inches long and increasing at a rate of .1 inches/second. How fast is the volume of the cube changing at this instant of time?

15. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of $2ft/s$, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?