Fall 2005 Math 151

courtesy: Amy Austin
(covering sections 5.3, 5.5)

Section 5.3

1. Verify $f(x) = x^2$ satisfies the Mean Value Theorem on the interval $[-1, 2]$ and find all c that satisfies the conclusion of the Mean Value Theorem.

2. Given that $f(1) = 10$ and $-2 \leq f'(x) \leq 2$ for all $x \in [1, 4]$, what are the largest and smallest possible values of $f(4)$?

3. Find the intervals where the given function is increasing or decreasing and identify all local extrema:

   a) $f(x) = 3x^4 + 4x^3 - 12x^2 + 8$

   b) $y = \tan^{-1}(x^2)$

   c) $f(x) = \dfrac{x}{(x-1)^2}$

   d) $f(x) = x \sin x + \cos x$ on $[-\pi, \pi]$

4. Determine the intervals where the given function is concave up or concave down and identify all inflection points for $f(x) = x^5 + 5x^4$

5. If $f'(x) = xe^{2x}$, find where $f(x)$ is increasing and decreasing, and locate intervals of concavity.

6. Given $f(-3) = 4$, $f'(-3) = 0$, $f''(-3) = 7$, $f(2) = -5$, $f'(2) = 0$, and $f''(2) = -6$, identify any local extrema of f.

Section 5.5

7. Postal regulations specify that a parcel sent by parcel post may have a combined length and girth of no more that 108 inches (see below). What are the dimensions of a package with a square front under the above guidelines which maximize the volume?

8. Find the point on the line $y = 2x - 3$ that is nearest to the origin.

9. A piece of wire 12 inches long is cut into two pieces. One piece is bent into an equilateral triangle and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a maximum? A minimum?

10. What are the dimensions of the largest rectangle that can be inscribed in the area bounded by the curve $y = 12 - x^2$ and the x-axis?