# Fall 2005 Math 152

**Night Before Drill**

*courtesy: Amy Austin*

(covering sections 8.4-9.6)

Review Exercises: Sections 8.4 - 9.6

## Section 8.4

1. $\displaystyle\int_2^3 \frac{dx}{(x-1)(x+2)}$
2. $\displaystyle\int \frac{dx}{x^2(x-1)}$
3. $\displaystyle\int \frac{5x^2-3x+4}{(x-1)(x^2+1)}\,dx$
4. $\displaystyle\int \frac{x^3}{x^2-x}\,dx$

## Section 8.8

5. Find the Midpoint Rule approximation $M_4$ to $\displaystyle\int_{-1}^{1} e^{x^2}\,dx$. Construct approximating rectangles.
6. Given the table below, use the Trapezoid rule with $n = 5$ to approximate $\displaystyle\int_1^2 f(x)\,dx$.

| x | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|---|
| f(x) | 4.8 | 5.4 | 5.8 | 6.2 | 6.8 | 7.0 |

7. a.) Use Simpson's rule with $n = 4$ to approximate $\displaystyle\int_1^2 \ln x\,dx$.

   b.) Use the error bound formula to find an upper bound on the error in using $S_4$ to approximate $\displaystyle\int_1^2 \ln x\,dx$.

   c.) How large do we need to choose $n$ so that the approximation $S_n$ to the integral $\displaystyle\int_1^2 \ln x\,dx$ has error less than $10^{-5}$?

## Section 8.9

8. Determine whether the following integrals converge or diverge. Evaluate those that converge.

   (a) $\displaystyle\int_0^\infty e^{-2x}\,dx$

   (b) $\displaystyle\int_{-2}^2 \frac{1}{x^2}\,dx$

   (c) $\displaystyle\int_0^1 \frac{x}{(x^2-1)^2}\,dx$

9. For each of the following integrals, determine whether the integral converges or diverges using the comparison theorem.

   (a) $\displaystyle\int_1^\infty \frac{\sin^4 x}{x^4}\,dx$

   (b) $\displaystyle\int_4^\infty \frac{x}{\sqrt{x^3-5}}\,dx$

   (c) $\displaystyle\int_0^{\pi/2} \frac{1}{x\sin x}\,dx$

## Sections 9.1 and 9.2

10. $xy' - 2y = 2x^4$, $y(1) = 0$
11. $\displaystyle y\frac{dy}{dx} = \frac{x^2}{e^{y^2}}$
12. $\displaystyle y' = \frac{e^{x-y}}{1+e^x}$, $y(1) = 0$
13. $y' + 2xy = xe^{-x^2}$
14. Given that $\displaystyle\frac{dt}{dx} = \frac{2x}{t^2}$ and $t(3) = 3$, find $t(1)$.
15. A curve passes through the point $(7, 4)$ and has the property that the slope of the curve at every point $P$ is 5 times the $y$-coordinate of $P$.What is the equation of the curve?
16. A tank contains 1000 liters of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much salt is in the tank after 25 minutes?

## Section 9.3

17. Find the length of the curve

    $x = \sin(2t)$, $y = \cos(2t)$, $0 \le t \le \dfrac{\pi}{4}$.

18. Find the length of the curve $\displaystyle x = \frac{2}{3}y^{\frac{3}{2}}$, $0 \le y \le 4$.

## Section 9.4

19. Find the surface area obtained by rotating the curve $y = x^2$, $0 \le x \le 2$ about the $y$ axis.

20. Find the integral that gives surface area obtained by rotating the curve $y = x^4$, $0 \leq x \leq 1$ about the $x$ axis. Do not evaluate the integral. Do the same for rotation around the $y$ axis.

21. The curve $y = \cos x$, $0 \leq x \leq \dfrac{\pi}{2}$ is rotated around the $x$ axis. Set up both a $dx$ and a $dy$ integral that gives the resulting surface area. Do not evaluate either integral.

22. Find the surface area obtained by rotating the curve $x = 3\sin t$, $y = 3\cos t$, $0 \leq t \leq \dfrac{\pi}{4}$ around the $y$ axis.

## Section 9.5

**Note: In this section, you will only be held responsible for the $x$-coordinate of the centriod.**

23. Find the center of mass of the system consisting of the masses 2 g, 5 g, 3 g, and 2 g located at the points $x = -10$, $x = 1$, $x = 2$, and $x = 6$ along the $x$ axis, respectively.

24. Find the moment about the $x$ axis, the moment about the $y$-axis, and the centroid of the system consisting of the masses 1 g, 3 g and 7 g located at the points $(-2, 3)$, $(5, 1)$ and $(6, -3)$, respectively.

25. Let $R$ be the region bounded by bounded by $y = x^4 - 1$, $y = 0$, $x = 1$, $x = 2$. Assume the uniform density of $R$ is $\rho = 9$. Find the moment of the system about the $y$-axis and the moment of the system about the $x$-axis. Also, find the centriod of $R$.

26. Find the centroid of the region bounded by

    $y = \cos x$, $y = \sin x$, $0 \leq x \leq \dfrac{\pi}{4}$.

## Section 9.6

27. A rectangular pool 20 meters long, 15 meters wide and 4 meters deep is filled with a fluid of density 1020 $kg/m^3$ to a depth of 3 meters.

    a.) Find the hydrostatic force on the bottom of the pool.

    b.) Find the hydrostatic force on the end of the pool.

28. A tank contains water. The ends of the tank are vertical and have the shape described below. Find the hydrostatic force against the end of the tank.

    a.) Semicircle of diameter 6 feet, with the diameter at the top. Assume the tank is full. The weight density of water is 62.5 pounds per cubic foot.

    b.) Isosceles triangle with height 4 feet, base length 2 feet with vertex at the top. The water level is 3 feet deep, measuring from the bottom. The weight density of water is 62.5 pounds per cubic foot.

    c.) The region bounded by $y = x^2$, $y = 4$.

29. A 2x2x2 $m^3$ cube is resting flat on its side at the bottom of a tank full of water to a depth of 20 m. The density of water is 1000 $kg/m^3$.

    a.) Find the force of water on the top of the cube.

    b.) Find the force of water on any one of the four vertical exposed sides.