Fall 2005 Math 152

Night Before Drill;
courtesy: Amy Austin

Review Exercises: Sections 10.1 - 10.9

Section 10.1

1. Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. If the sequence diverges, explain why.

   a.) $a_n = \dfrac{5\cos n}{n}$

   b.) $a_n = 3 + \dfrac{(-1)^n}{n}$

   c.) $a_n = \arccos\left(\dfrac{(n+1)!}{(n+3)!}\right)$

   d.) $a_n = \dfrac{(\arctan n)^5}{n^2}$

Section 10.2

2. Find the sum of the following series.

   a.) $\sum_{n=1}^{\infty}\left(\sin\dfrac{1}{n} - \sin\dfrac{1}{n+2}\right)$

   b.) $\sum_{n=1}^{\infty}\dfrac{1}{n(n+1)}$

   c.) $\sum_{n=1}^{\infty} 5\left(\dfrac{2}{7}\right)^n$

   d.) $\sum_{n=0}^{\infty}\dfrac{(-1)^{n+1}+3^n}{4^n}$

   e.) $\sum_{n=0}^{\infty}\dfrac{(-1)^n 3^{2n}}{5^{n+1}}$

   f.) $5 - \dfrac{5}{2} + \dfrac{5}{4} - \dfrac{5}{8} + \dfrac{5}{16} - ... + ...$

3. If the *nth* partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \dfrac{n-1}{n+1}$, find a_n and the sum of the series $\sum_{n=1}^{\infty} a_n$.

Section 10.3 and 10.4

4. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

   a.) $\sum_{n=2}^{\infty}\dfrac{1}{n\sqrt{\ln n}}$

   b.) $\sum_{n=2}^{\infty}\dfrac{\sqrt{n}}{n+1}$

   c.) $\sum_{n=1}^{\infty}\dfrac{(n+1)(-3)^n}{\sqrt{n}4^n}$

   d.) $\sum_{n=2}^{\infty}\dfrac{(-10)^{n-1}n^2}{(2n-3)!}$

   e.) $\sum_{n=1}^{\infty}\dfrac{(-1)^n n^2}{n^2+1}$

   f.) $\dfrac{3}{5} - \dfrac{3}{6} + \dfrac{3}{7} - \dfrac{3}{8} + \dfrac{3}{9} - ... + ...$

5. Determine whether the following series converge or diverge. For those that converge, determine whether they are absolutely convergent.

   a.) $\sum_{n=2}^{\infty}\dfrac{(-1)^n}{3n-1}$

   b.) $\sum_{n=1}^{\infty}\dfrac{(-1)^n n}{n^3+1}$

   c.) $\sum_{n=1}^{\infty}\dfrac{2}{n^3}$

   d.) $\sum_{n=1}^{\infty}\dfrac{(-10{,}000)^n n!}{(2n+1)!}$

6. Use S_{10} to estimate the sum of the series $\sum_{n=2}^{\infty}\dfrac{1}{n(\ln n)^3}$. Give an estimate on the remainder.

7. Consider the series $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n^5}$.

   a.) Use the first 5 terms to estimate the sum.

   b.) Estimate the error in the approximation s_5 to the sum of the series.

8. How many terms of the series do we need to add in order to find the sum to the indicated accuracy? $\sum_{n=0}^{\infty}\dfrac{(-1)^n}{n!}$, (error $< \dfrac{1}{100}$).

Section 10.5

9. For the following power series, find the radius and interval of convergence.

   a.) $\sum_{n=0}^{\infty}\dfrac{(\pi x)^n}{n^2+2}$

b.) $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{n!4^n}$

c.) $\sum_{n=1}^{\infty} \frac{(2n-1)!(x+2)^{n-1}}{5^{n-1}}$

d.) $\sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{\sqrt{n}}$

10. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ is convergent when $x = 4$ and divergent when $x = 6$. What can be said about the convergence or divergence of the following series:

a.) $\sum_{n=0}^{\infty} c_n 8^n$

b.) $\sum_{n=0}^{\infty} c_n(-3)^n$

c.) $\sum_{n=0}^{\infty} c_n(-4)^n$

d.) $\sum_{n=0}^{\infty} c_n(5)^n$

Section 10.6

11. Find a power series for the following functions and find the corresponding radius of convergence.

a.) $f(x) = \frac{1}{1-x}$

b.) $f(x) = \frac{1}{4+x^2}$

c.) $f(x) = \ln(2-x)$

d.) $f(x) = \frac{\arctan x}{x}$

e.) $f(x) = \frac{x}{(x^2+1)^2}$

12. Write $\int_0^{0.5} \frac{1}{1+x^4}\,dx$ as an infinite series.

Section 10.7

13. Find the Taylor Series for $f(x) = e^{4x}$ at $x = -1$.

14. Find the first three nonzero terms of the Taylor Series for $f(x) = \frac{1}{x^2}$ centered at $x = 3$.

15. Find a Maclaurin series for $\int \frac{\cos(x^2)}{x}\,dx$

16. Express $\int_0^1 e^{-x^2}\,dx$ as an infinite series. Use the first 2 terms of this series to approximate the sum and estimate the error.

17. Find the coefficient of $(x-4)^3$ in the Taylor Series for the function $f(x) = \sqrt{x}$ at $a = 4$.

18. Consider $f(x) = \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$. This function can be expressed as a series, by using the known Maclaurin series for $\sin x$. Find the first four nonzero terms of this series.

Section 10.9

For the problems 16-17, answer the following questions:

a.) Find $T_n(x)$ at the given value of a. Also, find the remainder term.

b.) Use Taylor's Inequality to estimate the accuracy of the approximation $T_n(x)$ for x in the given interval.

19. $f(x) = \frac{2}{x^2}$, $n = 4$, $a = 2$, $1 \le x \le 2.5$

20. $f(x) = \sqrt{1+x}$, $n = 2$, $a = 1$, $0 \le x \le 1.1$