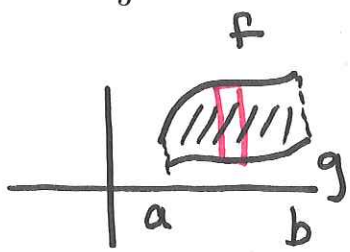
Spring 2013 Math 152

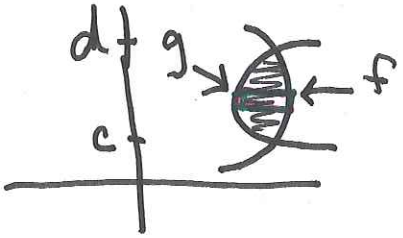
Week in Review 2
courtesy: Amy Austin
(covering section 7.1-7.2)

Section 7.1

1. Find the area bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$.

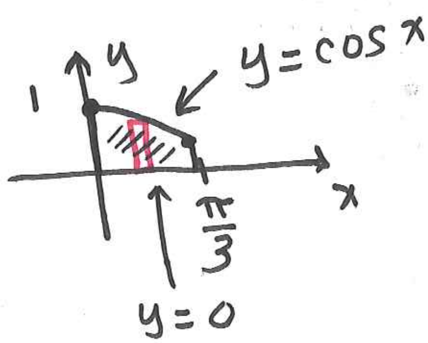


$$A = \int_a^b [f - g]\,dx$$



$$A = \int_c^d (f - g)\,dy$$

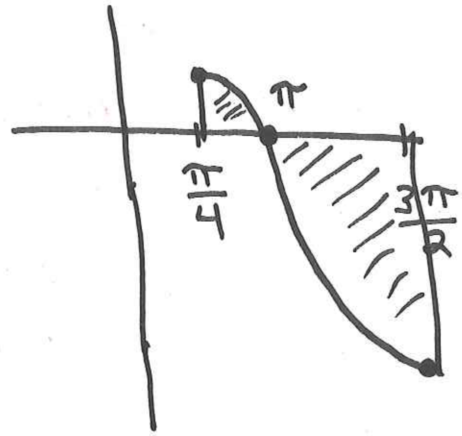
#1.



$$A = \int_0^{\frac{\pi}{3}} (\cos x - 0)\,dx$$

$$A = \sin x \Big|_0^{\frac{\pi}{3}} = \boxed{\frac{\sqrt{3}}{2} - 0}$$

2. Find the area bounded by $y = \sin x$, $y = 0$, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{2}$.



$$A = \int_{\frac{\pi}{4}}^{\pi} \sin x\,dx + \int_{\pi}^{\frac{3\pi}{2}} -\sin x\,dx$$

$$A = -\cos x \Big|_{\frac{\pi}{4}}^{\pi} + \cos x \Big|_{\pi}^{\frac{3\pi}{2}}$$

$$A = 1 - \left(-\frac{\sqrt{2}}{2}\right) + 0 - (-1)$$

$$\boxed{A = 2 + \frac{\sqrt{2}}{2}}$$

3. Find the area bounded by $y = x^2$ and $y = 2x - x^2$.

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x=0, \quad x=1$

$y=0, \quad y=1$

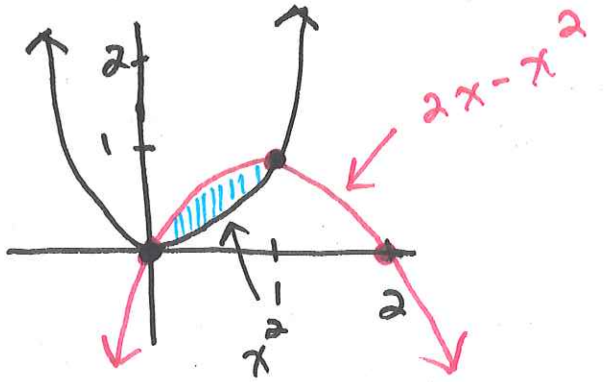
vertex of $2x-x^2$

$m=0$

$2-2x=0$

$x=1$

$y=1$



$$A = \int_0^1 \left(2x - x^2 - (x^2)\right) dx$$

$$A = \int_0^1 \left(2x - 2x^2\right) dx$$

$$= \left(x^2 - \frac{2}{3}x^3\right) \Big|_0^1$$

$$= 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

4. Find the area bounded by $y = x-1$ and $y^2 = 2x+6$.

$x = y+1 \leftarrow$ line

$x = \dfrac{y^2-6}{2} \leftarrow$ sideways parabola

$y+1 = \dfrac{y^2-6}{2}$

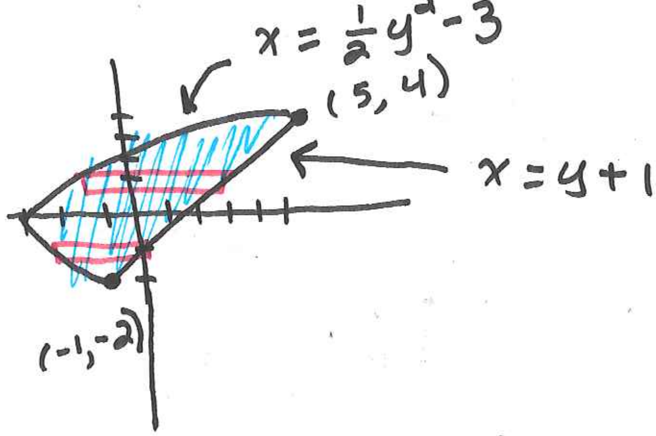
$2y+2 = y^2-6$

$0 = y^2 - 2y - 8$

$0 = (y-4)(y+2)$

$y=4, \quad y=-2$

$x=5, \quad x=-1$



$$A = \int_{-2}^{4} \left[y + 1 - \left(\frac{1}{2}y^2 - 3\right)\right] dy$$

$$\boxed{A = 18}$$

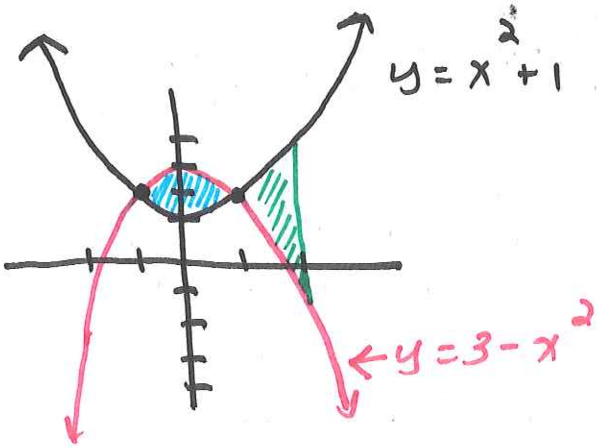
5. Find the area bounded by $y = x^2 + 1$, $y = 3 - x^2$, $x = -1$, $x = 2$.

$$x^2 + 1 = 3 - x^2$$

$$2x^2 = 2$$

$$x = 1,\ y = 2$$

$$x = -1,\ y = 2$$



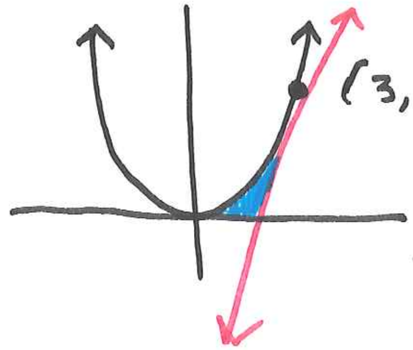
$$A = \int_{-1}^{1} \left[3 - x^2 - (x^2 + 1)\right] dx \ +$$

$$\int_{1}^{2} \left[x^2 + 1 - (3 - x^2)\right] dx$$

$$A = \int_{-1}^{1} (2 - 2x^2)\, dx + \int_{1}^{2} (2x^2 - 2)\, dx$$

$$\boxed{A = \frac{16}{3}}$$

6. Find the area of the region bounded by the parabola $y = 2x^2$, the tangent line to this parabola at $(3, 18)$ and the x-axis.



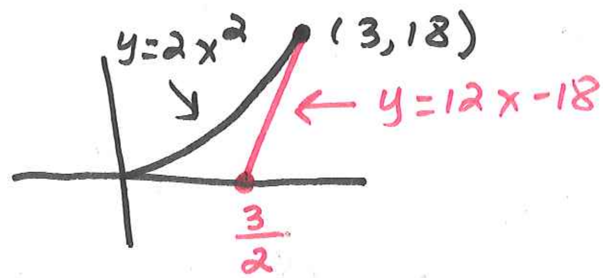
Tangent line to $y = 2x^2$ at $(3, 18)$:

$$y' = 4x$$

$$m = 12$$

equation: $y - 18 = 12(x - 3)$

$$y = 12x - 18$$



① $A = \int_{0}^{\frac{3}{2}} 2x^2\, dx + \int_{\frac{3}{2}}^{3} \left(2x^2 - (12x - 18)\right)$

② $A = \int_{0}^{18} \left(\frac{y + 18}{12} - \sqrt{\frac{y}{2}}\right) dy$

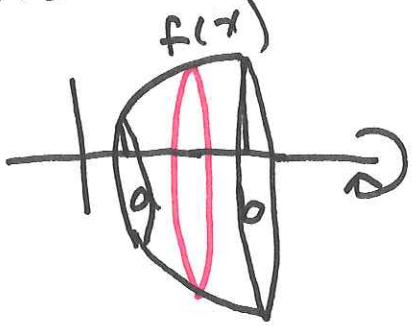
$$= \int_{0}^{18} \left(\frac{y + 18}{12} - \frac{1}{\sqrt{2}}\sqrt{y}\right) dy$$

$$= 4.5$$

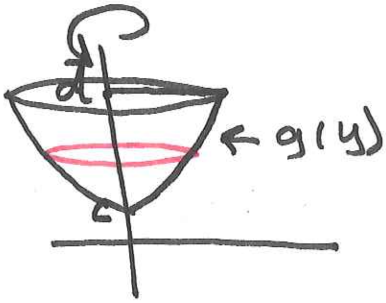
Section 7.2

7. Find the volume of the solid obtained by revolving the region bounded by $y = e^x$, $y = 0$, $x = 0$, $x = 1$ about the x-axis.

disk method:

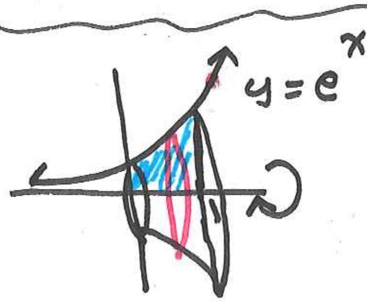


$$V = \int_a^b \pi (f(x))^2 \, dx$$



$$V = \int_c^d \pi (g(y))^2 \, dy$$

#7.

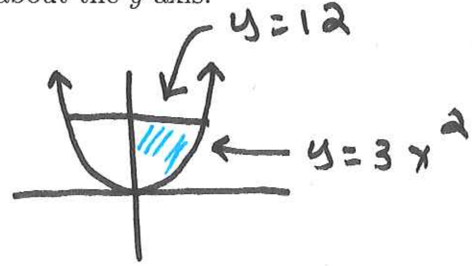


$$V = \int_0^1 \pi (e^x)^2 \, dx$$

$$V = \pi \int_0^1 e^{2x} \, dx$$

$$= \pi \cdot \frac{1}{2} e^{2x} \Big|_0^1 = \boxed{\frac{\pi}{2}(e^2 - 1)}$$

8. Find the volume of the solid obtained by revolving the region bounded by $y = 3x^2$, $y = 12$, $x = 0$ about the y-axis.



$$V = \int_0^{12} \pi \left(\sqrt{\frac{y}{3}} \right)^2 dy$$

$$= \frac{\pi}{3} \int_0^{12} (\sqrt{y})^2 dy$$

$$= \frac{\pi}{3} \int_0^{12} y \, dy$$

$$= \frac{\pi}{3} \frac{y^2}{2} \Big|_0^{12}$$

$$= \frac{\pi}{6} (144)$$

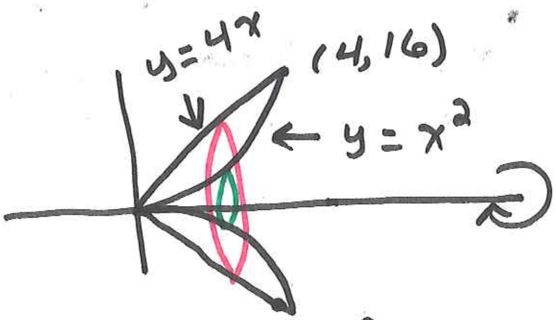
9. Find the volume of the solid obtained by revolving the region bounded by $y = x^2$, $y = 4x$, about the x-axis, then the y axis.

$$A = \pi R^2 - \pi r^2$$

$$A = \pi (R^2 - r^2)$$

$y = x^2$, $y = 4x$

about x-axis:

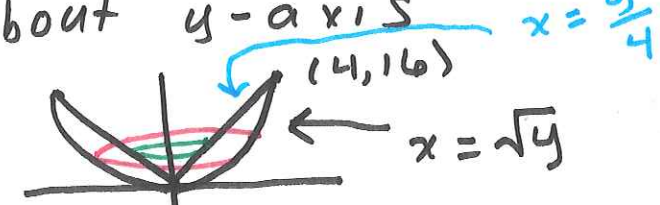


$R = 4x$, $r = x^2$

$$V = \int_0^4 \pi [(4x)^2 - (x^2)^2]\,dx$$

$$\boxed{V = \frac{2048\pi}{15}}$$

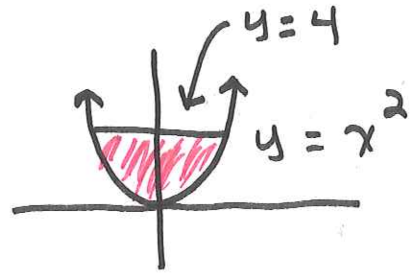
about y-axis

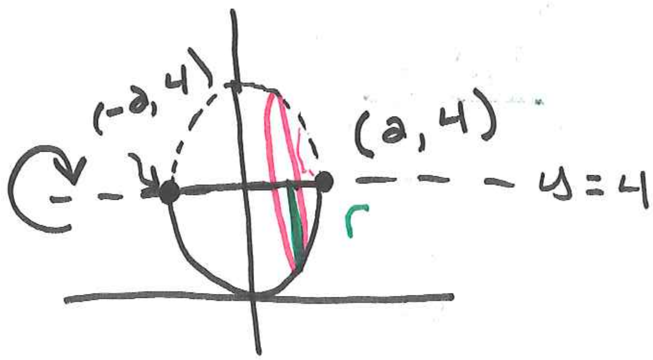


$R = \sqrt{y}$, $r = \frac{y}{4}$

$$V = \int_0^{16} \pi \left[(\sqrt{y})^2 - \left(\frac{y}{4}\right)^2 \right] dy = \frac{128\pi}{3}$$

10. Find the volume of the solid obtained by revolving the region bounded by $y = x^2$, $y = 4$, about the line $y = 4$.





$r = 4 - x^2$

$$V = \int_{-2}^{2} \pi (4 - x^2)^2\,dx$$

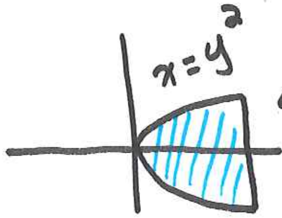
or

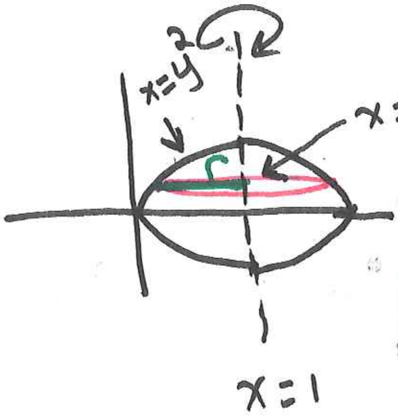
$$V = 2\int_0^2 \pi (4 - x^2)^2\,dx$$

symmetry

$$\boxed{V = \frac{512\pi}{15}}$$

11. Find the volume of the solid obtained by revolving the region bounded by $x = y^2$, $x = 1$, about the line $x = 1$.





$r = 1-y^2$

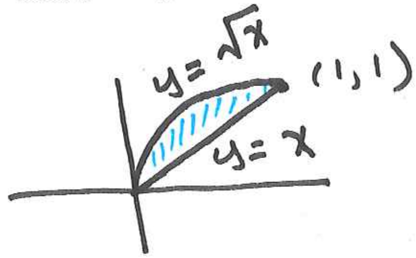
$V = \int_{-1}^{1} \pi (1-y^2)^2 \, dy$

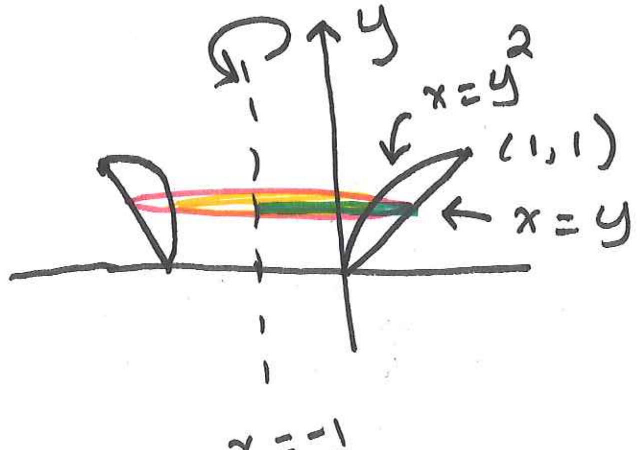
or

$V = 2\int_0^1 \pi (1-y^2)^2 \, dy$

$V = \frac{16\pi}{15}$

12. Find the volume of the solid obtained by revolving the region bounded by $y = x$, $y = \sqrt{x}$, about the line $x = -1$.





$R = y+1$

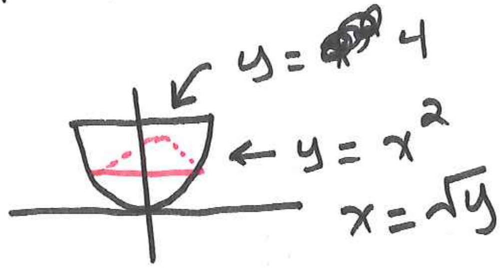
$r = y^2+1$

$V = \int_0^1 \pi \left[(y+1)^2 - (y^2+1)^2\right] dy$

$\boxed{V = \frac{7\pi}{15}}$

13. Find the volume of the solid S described here: The base of S is the region bounded by $y = x^2$ and $y = 4$. Cross-sections perpendicular to the y axis are equilateral triangles.

① draw the base of the solid. dy



② $V = \int_0^4 (A_{triangle})\,dy$

$A_{triangle} = \frac{1}{2} bh$

equilaterial triangle with side L, $h = \frac{\sqrt{3}}{2} L$

$b = 2\sqrt{y}$

$h = \frac{\sqrt{3}}{2}(2\sqrt{y}) = \sqrt{3}\sqrt{y}$

$A_{triangle} = \frac{1}{2}(2\sqrt{y})(\sqrt{3}\sqrt{y})$

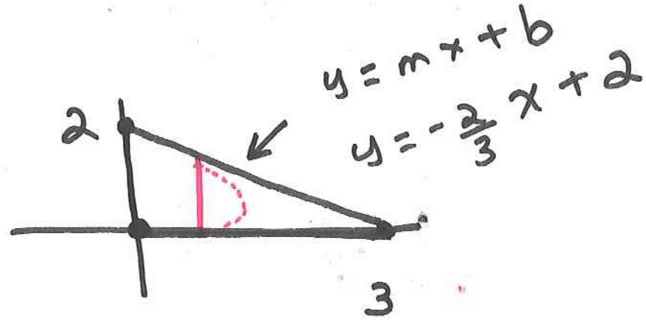
$= \sqrt{3}\, y$

$V = \int_0^4 \sqrt{3}\, y\, dy$

$= \sqrt{3} \frac{y^2}{2} \Big|_0^4 = \boxed{8\sqrt{3}}$

14. Find the volume of the solid S described here: The base of S is the triangular region with vertices $(0, 0)$, $(3, 0)$ and $(0, 2)$. Cross-sections perpendicular to the x axis are semi-circles.

① draw base:



② $V = \int_0^3 (A_{semicircle})\,dx$

$A_{semicircle} = \frac{1}{2}\pi r^2$

$d = -\frac{2}{3}x + 2$

$r = \frac{1}{2} d = \frac{1}{2}(-\frac{2}{3}x + 2)$

$r = -\frac{1}{3}x + 1$

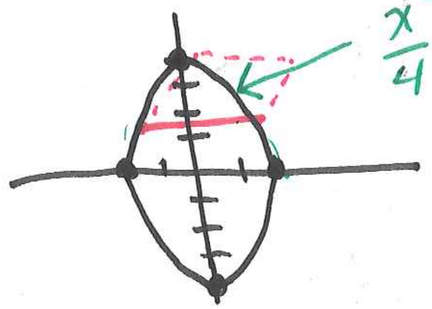
$A_{semicircle} = \frac{1}{2}\pi(-\frac{1}{3}x + 1)^2$

$V = \int_0^3 \frac{1}{2}\pi(-\frac{1}{3}x + 1)^2\, dx$

$V = \frac{13\pi}{27}$

15. Find the volume of the solid S described here: The base of S is the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$. Cross sections perpendicular to the y-axis are squares.

① base: $\frac{x^2}{4} + \frac{y^2}{16} = 1$



$$\frac{x^2}{4} + \frac{y^2}{16} = 1 \rightarrow \frac{x^2}{4} = 1 - \frac{y^2}{16}$$

$$x^2 = 4 - \frac{y^2}{4}$$

$$x = \sqrt{4 - \frac{1}{4}y^2}$$

②

$$V = \int_{-4}^{4} (A_{square})\, dy$$

$$A_{square} = (side)^2$$

$$side = 2\sqrt{4 - \frac{1}{4}y^2}$$

$$V = \int_{-4}^{4} \left(2\sqrt{4 - \frac{1}{4}y^2}\right)^2 dy$$

$$= 4\int_{-4}^{4} \left(4 - \frac{1}{4}y^2\right) dy$$

or by symmetry,

$$V = 8\int_{0}^{4} \left(4 - \frac{1}{4}y^2\right) dy$$