Fall 2005 Math 152

courtesy: Amy Austin
(covering sections 10.1, 10.2)

Section 10.1

1. Find the limit of the following sequences, if it exists. If the sequence diverges, state why.

   a.) $a_n = \dfrac{n}{\sqrt{n+2}}$

   b.) $a_n = \ln(n) - \ln(3n+1)$

   c.) $a_n = \dfrac{(-1)^n n}{n^2+1}$

   d.) $a_n = \dfrac{(-1)^n n^2}{n^2+1}$

   e.) $a_n = \dfrac{\ln n}{n}$

2. Suppose $\{a_n\}$ was given to be a convergent sequence, $a_1 = 2$, and $a_{n+1} = \dfrac{1}{3-a_n}$, find:

   a.) a_4

   b.) the limit of the sequence.

3. Determine whether the following sequences are increasing, decreasing, or non monotonic:

   a.) $a_n = \dfrac{1}{n^5}$

   b.) $a_n = \dfrac{n^2+4n+5}{n^2}$

   c.) $a_n = \dfrac{\ln n}{n}$

   d.) $a_n = \cos(n\pi)$

Section 10.2

4. Find the first few partial sums of the series $\sum_{n=1}^{\infty} \dfrac{1}{n}$ and $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$. Try to determine whether they converge/diverge.

5. Suppose $\sum_{n=1}^{\infty} a_n$ is a convergent series and $s_n = 5 + \dfrac{n}{2n+3}$ is a formula for the nth partial sum. What is the sum of the series?

6. Find the sum of the following series. If it diverges, support your answer.

   a.) $\sum_{n=1}^{\infty} \left(\dfrac{1}{n+5} - \dfrac{1}{n+6}\right)$

   b.) $\sum_{n=2}^{\infty} \ln\left(\dfrac{n}{n+1}\right)$

   c.) $\sum_{n=1}^{\infty} \dfrac{1}{n(n+2)}$

   d.) $\sum_{n=1}^{\infty} 2\left(\dfrac{1}{7}\right)^{n-1}$

   e.) $\sum_{n=1}^{\infty} (-5)\left(\dfrac{2}{3}\right)^{n}$

   f.) $\sum_{n=0}^{\infty} \dfrac{(-1)^n + 3^n}{5^n}$

   g.) $\sum_{n=2}^{\infty} \dfrac{(-1)^n 2^n}{3^{n+1}}$

   h.) $\sum_{n=0}^{\infty} \dfrac{(-1)^n 3^{2n}}{7^{n+1}}$

   i.) $4 + \dfrac{8}{5} + \dfrac{16}{25} + \dfrac{32}{125} + ...$