

MATH 151
Spring 2015

SAMPLE EXAM II

To be worked Wed, 3/25, 7:30-9:30 pm BLOC 102

Part I - Multiple Choice.

1. Using differentials or a linear approximation, approximate $\sqrt{11}$.

- a) $\frac{8}{3}$ b) $\frac{11}{3}$ c) $\frac{23}{6}$ d) $\frac{21}{6}$ e) $\frac{10}{3}$

2. If $f(x) = 3x \cos^2(x^2)$, find $f'(0)$.

- a) 0 b) -3 c) 3 d) 1 e) -9

3. A particle is moving according to the equation of motion $f(t) = t^4 - 4t + 1$, where t is measured in seconds and $f(t)$ is measured in feet. What is the acceleration of the particle at the instant when the particle is at rest?

- a) $0 \frac{ft}{s^2}$ b) $0 \frac{ft}{s}$ c) $12 \frac{ft}{s^2}$ d) $12 \frac{ft}{s}$ e) $-12 \frac{ft}{s^2}$

4. Let $f(x) = (1 + x^2)^{\frac{3}{2}}$. Then $f''(0) =$

- a) 3 b) 0 c) 6 d) $\frac{3}{4\sqrt{2}}$ e) $\frac{3}{4}$

5. The function $f(x) = x^3 + 5x - 1$ is one-to-one. Let $g = f^{-1}$. Then $g'(5) =$

- a) 8 b) $\frac{1}{80}$ c) $\frac{8}{25}$ d) $\frac{1}{8}$ e) 80

6. Given the curve parametrized by $x = t^3 - 3t^2 - 9t + 1$, $y = t^3 + 3t^2 - 9t + 1$, at which point does the curve have a vertical tangent?

- a) $(1, -3)$ b) $(6, 12)$ c) $(-10, 6)$
d) $(-1, 3)$ e) $(1, 1)$

7. $\lim_{x \rightarrow 0} \frac{4 \cos x - 4 + 3 \sin x}{5x} =$

- a) $\frac{4}{5}$ b) $-\frac{4}{5}$ c) $\frac{3}{5}$ d) 1 e) 0

8. Find the slope of the line tangent to the curve given by $y^2 + xy = 8$ at the point $(-2, -2)$.

- a) -2 b) $-\frac{10}{3}$ c) $-\frac{1}{3}$ d) -3 e) 0

9. $\lim_{x \rightarrow 5^+} e^{x/(5-x)} =$

- a) 0 b) ∞ c) $-\infty$ d) 1 e) e

10. Let $f(x)$ be a differentiable function and let $g(x) = 3x^2 - 1$. Let $H(x) = f(g(x))$, the composition of f and g . If $f(0) = 1$, $f'(0) = -1$, $f(1) = 3$, $f'(1) = 2$, $f(2) = -1$, $f'(2) = 5$, find $H'(1)$.

- a) 30 b) 12 c) -6 d) 6 e) 5

11. $\lim_{x \rightarrow \infty} 3^{1-x} =$

- a) 0 b) ∞ c) $-\infty$ d) 1 e) 3

12. Find the domain of vector function $\mathbf{r}(t) = \left\langle \frac{t}{t^2 - 16}, \sqrt{t - 2} \right\rangle$.

- a) $(-\infty, -4) \cup (-4, 2) \cup (2, 4) \cup (4, \infty)$
b) $(-\infty, -4) \cup (4, \infty)$
c) $(-\infty, -4) \cup (-4, 2] \cup [2, 4) \cup (4, \infty)$
d) $[2, 4) \cup (4, \infty)$
e) $(2, 4) \cup (4, \infty)$

13. If $\mathbf{r}(t) = \langle \cos 3t, t \rangle$ is the position of an object at time t , find the acceleration of the object at time $t = \frac{\pi}{9}$.

- a) $\left\langle \frac{1}{2}, 0 \right\rangle$
b) $\left\langle -\frac{1}{2}, 0 \right\rangle$
c) $\left\langle -\frac{9}{2}, 0 \right\rangle$
d) $\left\langle \frac{9}{2}, 0 \right\rangle$
e) $\langle 3, 0 \rangle$

14. If $f(x) = e^{x \tan x}$, find $f'(x)$.

a) $f'(x) = e^{x \tan x}$

b) $f'(x) = \sec^2 x e^{x \tan x}$

c) $f'(x) = (\tan x + x \sec^2 x) e^{x \tan x}$

d) $f'(x) = (\tan x + x \sec x \tan x) e^{x \tan x}$

e) $f'(x) = x \tan x e^{x \tan x - 1}$

15. Find the equation of the tangent line to the graph of $x = e^{2t}$, $y = te^t$ at the point $(1, 0)$.

a) $y = 2x - 1$

b) $y = 4x - 4$

c) $y = \frac{1}{2}x - \frac{1}{2}$

d) $y = \frac{1}{3}x - \frac{1}{3}$

e) $y = x - 1$

16. Find the quadratic approximation for $f(x) = \frac{1}{x}$ at $x = 1$.

a) $x^2 - 3x + 3$

b) $x^2 - x + 2$

c) $x^2 - 2x + 1$

d) $x^2 + 4x + 5$

e) $x^2 + x - 3$

Part II - Work Out

17. The position of a particle at time t is given by $\mathbf{r}(t) = \left\langle \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right\rangle$. Find the velocity and speed of the particle when $t = 0$.
18. $f(x) = \begin{cases} bx^2 - 2ax + 5 & \text{if } x \leq 2 \\ ax - 6 & \text{if } x > 2 \end{cases}$,
- a.) What must be true for $f(x)$ to be continuous everywhere?
- b.) Find the values of a and b that make $f(x)$ differentiable everywhere, if possible. If not possible, explain why.
19. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute, how fast is the water level rising when the height of the water is 1 foot?
20. Given the equation $2e^{xy} = x + y$, find $\frac{dy}{dx}$ when $x = 0$ and $y = 2$.
21. Sketch the graph of $f(x) = x^2 + x$ and show there are two tangent lines to the graph of $f(x)$ that pass through the point $(2, -3)$. Find an equation of these tangent lines.
22. A camera is positioned 4000 feet from the base of a rocket launching pad. At a particular moment, the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.
- a) How fast is the distance from the camera to the rocket changing at that moment?
- b) If the camera is focused on the rocket, how fast is the camera's angle of elevation changing at that moment?