Fall 2006 Math 151
Final Exam Practice - Solutions *courtesy: Amy Austin*

Final Exam Practice: Sections 1.1 - 6.5

1. a.) $< -5, -7 >$

   b.) $< -1/\sqrt{10}, -3/\sqrt{10} >$

   c.) $153°$

   d.) Vector projection: $< 2/5, 6/5 >$;

   scalar projection: $\frac{-4}{\sqrt{10}}$

2. A vector equation: $< 1 + 2t, -2 + 10t >$;

   parametric equations: $x = 1 + 2t$, $y = -2 + 10t$

3. above force=25.88 N, below force=36.60 N

4. Formula to use: $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

   a.) $f'(x) = \frac{1}{2\sqrt{1+x}}$

   b.) $f'(x) = \frac{-1}{(x-3)^2}$

5. a.) ∞

   b.) $\frac{1}{4}$

   c.) $-\frac{1}{4}$

   d.) The limit does not exist because $\lim_{x\to 3^+} f(x) = 17$ and $\lim_{x\to 3^-} f(x) = 5$

   e.) -3

   f.) $-\frac{1}{2}$

6. a.) 3

   b.) -2

   c.) -5

   d.) The limit does not exist

   e.) Not continuous at $x = 3$ (not in domain), not continuous at $x = -1$, $x = 5$ and $x = 7$ (the limit does not exist). Not differentiable at $x = -1$, $x = 3$, $x = 5$ and $x = 7$ (not continuous implies not differentiable). Also not differentiable at $x = -4$ and $x = -6$ because of sharp corners.

7. a.) Not continuous at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist. Continuous for all other values of x.

   b.) Not continuous at $x = 1$ because $\lim_{x\to 1} f(x) = 1$, yet $f(1) = 4$.

   c.) $a = 6$, $b = -3$

8. $x + y + 1 = 0$, $11x - y = 25$

9. horizontal asymptote: $y = 0$, vertical asymptote: $x = 1$.

10. $m = -8$, equation: $y = -8x$

11. 4

12. $x > \ln 4$

13. $f^{-1}(x) = \ln \frac{x}{1-x}$

14. a.) $y' = \frac{1 - 3x^2y + 9x^2}{x^3 + 4y^3 - 1}$

    b.) $y' = \frac{\sin(x-y) + 2y - 4}{\sin(x-y) - 2x}$

15. a.) $f'(x) = \frac{12x^2 - 4x^4 - 16x}{(1-x^2)^2}$

    b.) $f'(t) = 3t^2\cos(1-t^2) + 2t^4\sin(1-t^2)$

    c.) $G'(x) = 12\tan^2(4x-1)\sec^2(4x-1)$

16. 56

17. $y - \ln 27 = (3 + \ln 27)(x - \ln 3)$

18. At $t = -1$: $y + 17 = 6(x+1)$; horizontal tangent: $y = 64$; verical tangent: $x = 0$

19. $-\frac{7}{4}$

20. $L(x) = \ln 2 + 1/2(x-2)$,
    $Q(x) = \ln 2 + 1/2(x-2) - 1/8(x-2)^2$. The linear and quadratic approximations are useful in approximating the function for x sufficiently close to a.

21. $\frac{dh}{dt} = \frac{49}{36\pi}$ cm/min

22. $5/3$ feet per second

23. 0.5 feet per second

24. $x = 2$

25. $x = \frac{2e}{1-e}$ Since this is not in the domain, there is no solution.

26. $t = 2/5$

27. $x = t, y = e + 2et$

28. $f'(x) = -\tan x$

29. $y' = y\left(\frac{\ln(1+x+3x^3)}{2\sqrt{x}} + \frac{\sqrt{x}(1+9x^2)}{1+x+3x^3}\right)$, where

    $y = (1+x+3x^3)^{\sqrt{x}}$

30. 7.284 minutes

31. $t = .944$ hours

32. Inc: $(3, \infty)$, Dec: $(-\infty, 3)$, Local Min: $(3, -33)$; Local Max: None; Concave up: $(-\infty, 0)$ and $(2, \infty)$, concave down: $(0, 2)$, points of inflection: $(0, -6)$ and $(2, -22)$

33. $(0, \infty)$

34. Absolute Max: -1; Absolute min: -5

35. critical values: $x = -1$, $x = 1$, $x = 5$, f inc: $(-1, 1)$, $(5, \infty)$, ; f dec: $(-\infty, -1)$, $(1, 5)$; local min: $x = -1$,$x = 5$; local max: $x = 1$; f cu: $(-\infty, 0)$ and $(5, \infty)$; f cd: $(0, 4)$; inflection points: $x = 0$, $x = 5$.

36. a.) e^3

    b.) 0

    c.) 0

37. $2/5x^{5/2} - 4/3x^{3/2} + 2x^{1/2}$

38. 1.448745691

39. $5/\sqrt{29}$

40. $\frac{\pi}{3}$

41. $y' = -\frac{3}{\sqrt{1-9t^2}} - \frac{1}{\sqrt{t}(1+t)}$

42. $2\sqrt{30}$ by $\frac{90}{\sqrt{30}}$

43. 5 feet by 10 feet long

44. $2x\sqrt{1-x^8}$

45. $-\frac{5}{9(3x^3-1)} + C$

46. $\frac{e^{-3} - e^{-5}}{6}$

47. -2

48. $\ln|t^2 - 1| + C$

49. $\frac{1}{\pi}$

50. $140/3 - 4\ln 4$

51. $-1/3\ln|\cos 3x|$+C

52. $2.5\ln 5$

53. $\frac{1}{4\cos^4 x} + C$

54. $2/3(x+1)^{1.5} - 2\sqrt{x+1} + C$