Spring 2012 Math 152

Overview of Material for Test I

courtesy: Amy Austin

The Fundamental Theorem of Calculus (6.4)

1. Part I: If f is continuous on $[a, b]$, then the function g defined by

   $g(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

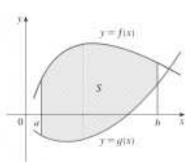
2. Part II: If f is continuous on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is an antiderivative of f.

The substitution rule (6.5):

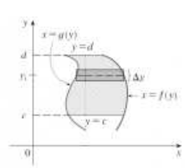
3. If $u = g(x)$ is a differentiable function, then $\int f(g(x))g'(x)\,dx = \int f(u)\,du$

Area (7.1)

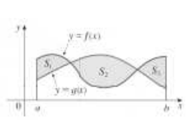
4. a.) $A = \int_a^b [Top - Bottom]\,dx$ if an x-integral is preferred.



b.) $A = \int_c^d [Right - Left]\,dy$ if a y-integral is preferred.



c.) If we are asked to find the area bounded by the curves $y = f(x)$, $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x, we must split the integral at each intersection point.

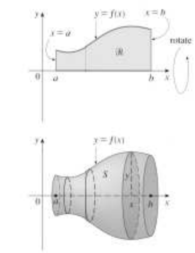


Volume (7.2-7.3)

5. a.) **Disk**: Note: The formulas below are for the general case. See my lecture notes for variations.

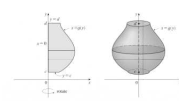
- $V = \int_a^b \pi r^2\,dx$ if revolving around the x axis.

$r = f(x)$



- $V = \int_c^d \pi r^2\,dy$ if revolving around the y axis.

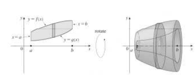
$r = g(y)$



b.) **Washer:** Note: The formulas below are for the general case. See my lecture notes for variations.

- $V = \int_a^b \pi(R^2 - r^2)\,dx$ if revolving around the x axis.

R = Top function; r =bottom function.



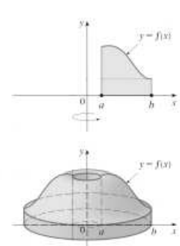
- $V = \int_c^d \pi(R^2 - r^2)\,dy$ if revolving around the y axis.

R = Right function; r =Left function.

c.) **Shell**: Note: The formulas below are for the general case. See my lecture notes for variations.

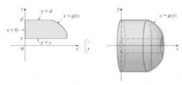
- $V = \int_a^b 2\pi rh\,dx$ if revolving around the y axis.

$r = x$, $h = f(x)$



• $V = \int_c^d 2\pi r h \, dy$ if revolving around the x axis.

$r = y$, $h = g(y)$



d.) **Slicing**:

• $V = \int_a^b A \, dx$ if cross sections are perpendicular to the x axis, and A is the area of a cross section.

• $V = \int_c^d A \, dy$ if cross sections are perpendicular to the y axis, and A is the area of a cross section.

Work (7.4)

6. If the force F is constant, then the work W done in moving the object a distance d is $W = Fd$.

7. If the force $f(x)$ is not constant, then the work W done in moving the object from $x = a$ to $x = b$ is $W = \int_a^b f(x) \, dx$.

a.) **Spring problems**: The force $f(x)$ needed to maintain a spring stretched x units beyond its *natural* length is $f(x) = kx$. Therefore the work W required to stretch the spring from $x = a$ to $x = b$ units beyond the natural length is $W = \int_a^b f(x) \, dx$.

b.) **Rope pulling problems**:

• Pulling the rope only: If the rope is b units long and weighs w (Pounds or Newtons) per unit of length, then the work required to pull the entire rope is $W = \int_0^b wx \, dx$.

• Pulling the rope with weight attached: The total work is the work required to pull the rope plus the work required to lift the weight. Note if we are only pulling a portion of the rope to the top, then we can think of the remaining rope as a weight that needs to be pulled to the top.

c.) **Water pumping problems**: There are many variations, depending on the shape of the tank, the shape of a horizontal cross section of water, whether the tank is full, and how much of the tank you are emptying. To be as general as possible, $W = \rho g \int dA \, dy$, where ρg is the weight density of water, A is the area of an arbitrary (horizontal) slice of water and d is the distance the slice of water must travel to exit the tank. Note if the tank has a spout on the top of the tank, then this affects the distance.

Average Value (7.5)

8. The average value of $f(x)$ from $x = a$ to $x = b$ is $f_{ave} = \frac{1}{b-a} \int_a^b f(x) \, dx$.

The Mean Value Theorem for Integrals states that if $f(x)$ is continuous over the interval $[a, b]$, then there is a number c, $a \leq c \leq b$ so that $f(c) = f_{ave}$.

Integration by Parts (8.1)

9. Formula: $\int u \, dv = uv - \int v \, du$

Case I:

$\int x^n e^{kx} \, dx$: $u = x^n$; $dv = e^{kx} dx$

$\int x^n \sin(kx) \, dx$: $u = x^n$; $dv = \sin(kx) dx$

$\int x^n \cos(kx) \, dx$: $u = x^n$; $dv = \cos(kx) dx$

Case II:

$\int x^n (\ln x) \, dx$: $u = \ln x$; $dv = x^n dx$

Case III:

$\int x^n \arccos x \, dx$: $u = \arccos x$; $dv = x^n dx$

$\int x^n \arcsin x \, dx$: $u = \arcsin x$; $dv = x^n dx$

$\int x^n \arctan x \, dx$: $u = \arctan x$; $dv = x^n dx$

Case IV:

$\int e^x \cos x \, dx$: $u = e^x$; $dv = \cos x \, dx$

$\int e^x \sin x \, dx$: $u = e^x$; $dv = \sin x \, dx$

Trig Integrals (8.2)

10. Integrals of the form $\int \sin^m x \cos^n x \, dx$:

    a.) If m is odd (and positive), factor out one sine and use
    $\sin^2 x = 1 - \cos^2 x$. Then, substitute $u = \cos x$.

    b.) If n is odd (and positive), factor out one cosine and use $\cos^2 x = 1 - \sin^2 x$. Then, substitute $u = \sin x$.

    c.) If both m and n are odd, use either case above (but not both).

    d.) If both m and n are even, use the identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

11. Integrals of the form $\int \tan^m x \sec^n x \, dx$:

    a.) If m is odd (and positive), factor out one $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$. Then, substitute $u = \sec x$.

    b.) If n is even (and positive), factor out one $\sec^2 x$ and use the identity $\sec^2 x = 1 + \tan^2 x$. Then, substitute $u = \tan x$.

    c.) If m is odd and n is even, use either case above (but not both).

    d.) If m is even and n is odd, try breaking up into sines and cosines.

12. Integrals of the form [The identities below need not be memorized for the exam]

    - $\int \sin(Ax)\cos(Bx)\,dx$: Use the identity $\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$
    - $\int \sin(Ax)\sin(Bx)\,dx$: Use the identity $\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$
    - $\int \cos(Ax)\cos(Bx)\,dx$: Use the identity $\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$