1. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

Test for divergence: (T.D.)

If $\lim_{n\to\infty} a_n \neq 0$, $\sum_{n=1}^{\infty} a_n$ diverges

a.) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

$\lim_{n\to\infty} \frac{1}{n\sqrt{\ln n}} = 0$ T.D. Fails

$\frac{1}{n\sqrt{\ln n}} > 0$ and decreasing and integratable

integral test $\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int \frac{du}{\sqrt{u}} = 2u^{\frac{1}{2}}$

$= 2\sqrt{\ln x}$

$= 2\sqrt{\ln x}\Big|_2^{\infty}$

$= 2\cancel{\sqrt{\ln \infty}}^{\infty} - 2\sqrt{\ln 2}$

$\boxed{\text{integral diverges, so does the series.}}$

b.) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n+1}}$

$\boxed{\text{T.D. } \lim_{n\to\infty} \frac{n}{\sqrt{n+1}} = \infty \neq 0 \text{ series diverges by T.D.}}$

T.D. fails

CT: comparison test can only be used on series with <u>positive terms</u>

"If larger series converges, so does the smaller. If smaller series diverges, do does the larger"

c.) $\sum_{n=2}^{\infty} \frac{2+\cos n}{n^3+n^2+1}$

$\leq \sum_{n=2}^{\infty} \frac{2+1}{n^3} = \sum_{n=2}^{\infty} \frac{3}{n^3}$ convergent p-series $p = 3 > 1$

larger series converges, so does the smaller by C.T.

d.) $\sum_{n=2}^{\infty} \frac{n}{n^2+n+1}$

T.D. fails Try C.T.

$\sum_{n=2}^{\infty} \frac{n}{n^2+n+1} \leq \sum_{n=2}^{\infty} \frac{n}{n^2} = \sum_{n=2}^{\infty} \frac{1}{n}$

divergent p-series $p=1$

Larger series diverges

CT fails.

Limit comparison Test LCT

only can be used on positive series.

$\lim_{n\to\infty} \frac{a_n \leftarrow \text{original}}{b_n \leftarrow \text{comparison}} = c > 0$ & finite then $\sum_{n=1}^{\infty} a_n$ & $\sum_{n=1}^{\infty} b_n$ behave the same way

LCT $\lim_{n\to\infty} \left(\frac{\frac{n}{n^2+n+1}}{\frac{1}{n}} \right) = \lim_{n\to\infty} \frac{n^2}{n^2+n+1}$

$= 1 > 0$

because $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=2}^{\infty} \frac{n}{n^2+n+1}$ by LCT

2. Consider the series $\sum_{n=1}^{\infty} ne^{-n^2}$. Prove the series converges using the Integral Test. Use S_6 to estimate the sum of the series and estimate the remainder (error).

$f(x) = xe^{-x^2}$ positive & decreasing

I.T. $\int_1^\infty xe^{-x^2}\,dx$ $\quad u = -x^2$, $du = -2x\,dx$ $\quad -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u = -\frac{1}{2}e^{-x^2}$

$$= -\frac{1}{2}e^{-x^2}\Big|_1^\infty$$

$$= -\frac{1}{2}\left[\cancel{e^{-\infty}}^{\,0} - e^{-1}\right]$$

$$= \frac{1}{2e} < \infty$$

integral converges so does the series

$$\sum_{n=1}^{\infty} ne^{-n^2} \approx S_6 = \sum_{n=1}^{6} ne^{-n^2}$$

$$\boxed{S_6 = e^{-1} + 2e^{-4} + 3e^{-9} + \dots + 6e^{-36}}$$

Remainder formula for series of positive terms is $R_{\boxed{n}} = S - S_n \le \int_{\boxed{n}}^\infty f(x)\,dx$

estimate on the error [aka remainder] is

$$R_6 \le \int_6^\infty xe^{-x^2}\,dx = -\frac{1}{2}e^{-x^2}\Big|_6^\infty$$

$$= -\frac{1}{2}\left[\cancel{e^{-\infty}}^{\,0} - e^{-36}\right]$$

$$\boxed{R_6 \le \frac{1}{2e^{36}}}$$

3. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ T.D. Fails see "$(-1)^n$" do AST

show $\left\{ \frac{1}{\ln n} \right\}$ decreases to zero.
①

① verify $a_{n+1} \leq a_n$

Both met

$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges by AST $\left\{ \begin{array}{l} \frac{1}{\ln(n+1)} \leq \frac{1}{\ln n} \quad \checkmark \\ \lim_{n\to\infty} \frac{1}{\ln n} = 0 \quad \checkmark \end{array} \right.$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n+1}}$ T.D. $\lim_{n\to\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n+1}} \neq 0$

$\boxed{\text{series diverges by T.D.}}$

4. Determine whether the following series diverge, coonverge absolutely, or converge conditionally (converges but not absolutely).

a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{3n-1}$

Def: $\sum_{n=1}^{\infty} a_n$ is absolutely convergent if $\sum_{n=1}^{\infty} |a_n|$ converges.

T.D. fails

Test for absolute convergence by testing

$$\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{3n-1} \right| = \sum_{n=2}^{\infty} \frac{1}{3n-1} \geq \sum_{n=2}^{\infty} \frac{1}{3n}$$

smaller diverges by p-series (p=1) so does larger by C.T.

$\sum_{n=2}^{\infty} \frac{(-1)^n}{3n-1}$ does not converge absolutely.

However, by AST it does converge since $\left\{ \frac{1}{3n-1} \right\}$ decreases to zero

$$\frac{1}{3(n+1)-1} \leq \frac{1}{3n-1} \checkmark$$

$$\lim_{n\to\infty} \frac{1}{3n-1} = 0 \checkmark$$

$\boxed{\text{series converges but not absolutely}}$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3+1}$

T.D. fails.

Test for absolute convergence by testing $\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{n^3+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^3+1}$

CT: $\sum_{n=1}^{\infty} \frac{n}{n^3+1} \leq \sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

larger convergent p-series, smaller converges.

$\boxed{\text{converges absolutely}}$

TD fails.

c.) $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{1}{n}\right)}{n^2}$

Test for absolute convergence

$$\sum_{n=1}^{\infty} \left| \frac{\cos \frac{1}{n}}{n^2} \right| \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$$

convergent p-series $p = 2 > 1$

$\boxed{\text{Series converges absolutely}}$

d.) $\sum_{n=1}^{\infty} \frac{(-10)^n n!}{(2n+1)!}$

Ratio Test $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

L>1 diverges

L<1 converges absolutely

L=1 test fails

$$\lim_{n\to\infty} \left| \frac{(-10)^{n+1}(n+1)!}{(2n+3)!} \cdot \frac{(2n+1)!}{(-10)^n n!} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-10)^n}(-10)(n+1)\cancel{n!}}{(2n+3)(2n+2)\cancel{(2n+1)!}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{(-10)^n}\cancel{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(-10)(n+1)}{(2n+3(2n+2)} \right| = 0 < 1$$

$\boxed{\text{series converges absolutely}}$

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$. converges absolutely because $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges

a.) Use the first 5 terms to estimate the sum.

* b.) Estimate the error in the approximation s_5 to the sum of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \approx S_5 = \sum_{n=1}^{5} \frac{(-1)^n}{n^5} = -1 + \frac{1}{2^5} - \frac{1}{3^5} + \frac{1}{4^5} - \frac{1}{5^5}$$

If $\sum_{n=1}^{\infty} (-1)^n b_n$ is a convergent alternating series, $b_n > 0$. Then

$$|R_n| \le |b_{n+1}|$$

Alternating series estimation theorem

$$|R_5| \le |a_6| = \frac{1}{6^5}$$

$\sum_{n=1}^{\infty} c_n (x-a)^n$ center is a.

Section 10.5

6. For the following power series, find the radius and interval of convergence.

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n \sqrt{n}}$ centered at -1.

$$\text{RT} \quad \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x+1)^{n+1}}{2^{n+1}\sqrt{n+1}} \cdot \frac{2^n \sqrt{n}}{(-1)^n (x+1)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)(x+1)}{2\sqrt{n+1}} \frac{\sqrt{n}}{1} \right|$$

$$= \left| -\tfrac{1}{2}(x+1) \right| < 1$$

$$= \tfrac{1}{2}|x+1| < 1 \qquad \boxed{R=2}$$

$$\boxed{|x+1| < 2}$$

$-3 \quad -1 \quad 1$

$2 \quad 2$

Test endpoints for convergence

Section 10.5

6. For the following power series, find the radius and interval of convergence.

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n \sqrt{n}}$

Test $x = -3$: $\sum_{n=1}^{\infty} \frac{(-1)^n (-2)^n}{2^n \sqrt{n}}$ 2^n

not included.

$= \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ divergent p-series $p = \frac{1}{2}$

Test $x = 1$: $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{2^n \sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by AST

included

$\left\{ \frac{1}{\sqrt{n}} \right\}$ decreases to zero show it!

$$\boxed{I = (-3, 1]}$$

b.) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$I = (-\infty, \infty)$

$R = \infty$

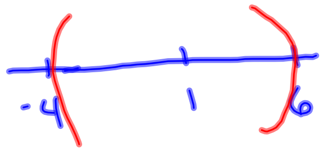
c.) $\sum_{n=1}^{\infty} \frac{(2n-1)!(x+2)^{n-1}}{5^{n-1}}$

$I = \{-2\}$

$R = 0$

7. If the series $\sum_{n=1}^{\infty} c_n(x-1)^n$ has a radius of convergence of 5, then what do we know about the following series:

$\sum_{n=1}^{\infty} c_n(3)^n$, $\sum_{n=1}^{\infty} c_n(5.5)^n$



$x = 4$ $\quad -4 < x < 6$ will converge

if $-4 < x < 6$ guaranteed convergence.

$x < -4$ & $x > 6$ divergence

$x = 6.5$ not between $-4 < x < 6$ will diverge.

Section 10.6 and 10.7

7. Find a Maclaurin series for the following functions and the associated radius of convergence.

a.) $f(x) = \dfrac{1}{8+3x^2}$

Note $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < \boxed{1}$ R

$$= \frac{1}{8\left(1+\frac{3x^2}{8}\right)} = \frac{1}{8}\,\frac{1}{1-\left(-\frac{3x^2}{8}\right)}$$

$$= \frac{1}{8}\sum_{n=0}^{\infty}\left(-\frac{3x^2}{8}\right)^n, \quad \left|-\frac{3x^2}{8}\right| < 1$$

$$|x| < \sqrt{\frac{8}{3}}$$

$$= \frac{1}{8}\sum_{n=0}^{\infty}\frac{(-3)^n x^{2n}}{8^n} \qquad \boxed{R = \sqrt{\frac{8}{3}}}$$

$$= \boxed{\sum_{n=0}^{\infty}\frac{(-3)^n x^{2n}}{8^{n+1}}}$$

b.) $f(x) = \dfrac{x}{(1-x^2)^2}$

$$\int \frac{x}{(1-x^2)^2}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$$-\frac{1}{2}\int\frac{du}{u^2}$$

$$-\frac{1}{2}\left(-\frac{1}{u}\right)$$

$$\frac{1}{2(1-x^2)}$$

$$\int \frac{x}{(1-x^2)^2}\,dx = \frac{1}{2}\left(\frac{1}{1-x^2}\right)$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}(x^2)^n, \quad |x^2| < 1$$

$$|x| < 1$$

$$\boxed{R = 1}$$

$$\int \frac{x}{(1-x^2)^2}\,dx = \frac{1}{2}\sum_{n=0}^{\infty} x^{2n}$$

$$\frac{x}{(1-x^2)^2} = \frac{d}{dx}\left[\frac{1}{2}\sum_{n=0}^{\infty} x^{2n}\right]$$

$$= \frac{1}{\cancel{2}}\sum_{n=1}^{\infty}(\cancel{2}n)\,x^{2n-1}$$

$$= \boxed{\sum_{n=1}^{\infty} n\,x^{2n-1}}$$

c.) $f(x) = \ln(2-x)$

$$\frac{d}{dx}\ln(2-x) = \frac{-1}{2-x}$$

$$= \frac{-1}{2\left(1-\frac{x}{2}\right)}$$

$$= -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n \qquad \left|\frac{x}{2}\right| < 1$$

$$|x| < 2$$

$$= -\frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{2^n}x^n \qquad \boxed{R=2}$$

$$\frac{d}{dx}\ln(2-x) = -\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}x^n$$

$$\ln(2-x) = \int -\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}x^n\,dx$$

$$\ln(2-x) = \overset{\ln 2}{\cancel{C}} - \sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\frac{x^{n+1}}{n+1}$$

let $x=0$ to find C: $\ln 2 = C - \sum 0$

d.) $f(x) = x^5 e^{8x^2}$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^{8x^2} = \sum_{n=0}^{\infty}\frac{(8x^2)^n}{n!} = \sum_{n=0}^{\infty}\frac{8^n x^{2n}}{n!}$$

$$x^5 e^{8x^2} = x^5\sum_{n=0}^{\infty}\frac{8^n x^{2n}}{n!}$$

$$= \sum_{n=0}^{\infty}\frac{8^n x^{2n+5}}{n!} \qquad \boxed{R=\infty}$$

e.) $f(x) = \sin\left(\frac{x}{3}\right)$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin\left(\frac{x}{3}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{3}\right)^{2n+1}}{(2n+1)!} \qquad \boxed{R = \infty}$$

f.) $\int x \arctan(x^3)\, dx$

Known: $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad \boxed{R=1}$

$$\arctan(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n+1}}{2n+1}$$

$$\arctan(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1}$$

$$x \arctan(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{2n+1}$$

$$\int x \arctan(x^3)\, dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+5}}{(2n+1)(6n+5)}$$

8. Consider the Taylor Series for $f(x) = \ln x$ centered at 2. What is the coefficient of $(x-2)^4$?

Taylor series for $f(x)$ about a is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\ln x = \sum_{n=0}^{\infty} \frac{f^{\boxed{n}}(2)}{\boxed{n!}}(x-2)^{\boxed{n}}$$

coefficient of $(x-2)^4$ is $\dfrac{f^4(2)}{4!}$

$$= \boxed{\frac{-\frac{3}{8}}{4!}}$$

$f = \ln x$ $\quad f'' = -\frac{1}{x^2}$ $\quad f^4 = -\frac{6}{x^4}$

$f' = \frac{1}{x}$ $\quad f''' = \frac{2}{x^3}$ $\quad f^4(2) = \frac{-6}{16} = \frac{-3}{8}$

9. Find the Taylor Series for $f(x) = (x+2)e^x$ at $x = 1$.

$$(x+2)e^x = \sum_{n=0}^{\infty} \frac{f^n(1)}{n!}(x-1)^n$$

$f = (x+2)e^x$

$f' = (x+2)e^x + e^x = (x+3)e^x$

$f'' = (x+3)e^x + e^x = (x+4)e^x$

$f''' = (x+4)e^x + e^x = (x+5)e^x$

$f^{(n)}(x) = (x+2+n)e^x$

$f^{(n)}(1) = (3+n)e$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n = \boxed{\sum_{n=0}^{\infty} \frac{(3+n)e}{n!}(x-1)^n}$$

10. Using the known Maclaurin for $\cos(x)$, find the 40^{th} derivative at $x = 0$ for $f(x) = \cos\left(\frac{x^2}{2}\right)$.

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos\left(\frac{x^2}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{2^{2n}(2n)!}$$

$f(x) = x^6$

$f^{6}(x) = 6!$

$$= a_1 + a_2 + \cdots + a_{10} + a_{11} + a_{12} + \cdots$$

$$\downarrow$$

$$\frac{(-1)^{10} x^{40}}{2^{20}(20)!}$$

40^{th} deriv is:

$$\frac{1}{2^{20}(20)!} 40!$$

11. Express $\int_0^1 e^{-x^2}\,dx$ as an infinite series. Use the first 2 terms of this series to approximate the sum.

$$\int_0^1 e^{-x^2}dx = \int_0^1 \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}\,dx$$

$$= \int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}\Bigg|_0^1$$

$$\int_0^1 e^{-x^2}dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!}$$

$$\int_0^1 e^{-x^2}dx \approx S_1 = \sum_{n=0}^{1} \frac{(-1)^n}{(2n+1)n!}$$

$$= \boxed{1 - \frac{1}{3}}$$

$$\boxed{|R_1| \leq |a_2| = \frac{1}{(5)2!}}$$

Section 10.9

12. If $f(x) = \sqrt{1+x}$, $n = 2$, $a = 3$, $2 \le x \le 3.1$

a.) Find $T_n(x)$ at the given value of a.

b.) Use Taylor's Inequality to estimate the accuracy of the approximation $T_n(x)$ for x in the given interval.

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{n}(a)(x-a)^n}{n!}$$

$a = 3, \quad n = 2$

$$T_2(x) = \overset{2}{\cancel{f(3)}} + \overset{\frac{1}{4}}{\cancel{f'(3)}}(x-3) + \frac{\overset{\frac{-1}{4(8)}}{\cancel{f''(3)}}}{2}(x-3)^2$$

$f(x) = \sqrt{x+1}$

$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+1}}$

$f''(x) = \frac{-1}{4}(x+1)^{-\frac{3}{2}} = -\frac{1}{4(x+1)^{3/2}}$

Taylor's inequality

$$|R_n| \le \frac{M}{(n+1)!}|x-a|^{n+1} \qquad M = \max|f^{n+1}(x)|$$

$a = 3$

$n = 2$

$M = \max|f'''(x)| \quad 2 \le x \le 3.1$

$f'''(x) = \frac{3}{8}(x+1)^{-\frac{5}{2}}$

$$M = \max\left|\frac{3}{8(x+1)^{5/2}}\right| \quad 2 \le x \le 3.1$$

$$\boxed{M = \frac{3}{8(3)^{5/2}}}$$

$$|R_2(x)| \le \boxed{\frac{\frac{3}{8(3)^{5/2}}}{3!}}\,|\underbrace{x-3}_{1}|^3$$

1 = max distance

2 — 3 — 3.1

Section 11.1 and 11.2

13. Given the points $A(5,5,1)$, $B(3,3,2)$ and $C(1,4,4)$, determine whether triangle ABC is isosceles, right, both, or neither. Also, find the angle located at A.

$|AB| = \sqrt{4+4+1} = 3$

$|BC| = \sqrt{4+1+4} = 3$

$|AC| = \sqrt{16+1+9} = \sqrt{26}$

since $|AB| = |BC|$, ΔABC is isosceles.

since ΔABC does not satisfy pyth. theorem, it is not right.

$$\cos\alpha = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{\langle -2,-2,1\rangle\cdot\langle -4,-1,3\rangle}{\sqrt{4+4+1}\ \sqrt{16+1+9}} = \frac{13}{\sqrt{9}\sqrt{26}}$$

$$= \frac{13}{3\sqrt{2\cdot 13}}$$

$$\cos\alpha = \frac{13}{3\sqrt{13}\sqrt{2}}$$

$$\alpha = \arccos\left(\frac{\sqrt{13}}{3\sqrt{2}}\right)$$

14. Find the center and radius of the sphere $x^2+y^2+z^2+x+2y-2=0$. What is the intersection of this sphere with the xz plane?

$$x^2 + x + \frac{1}{4} + y^2 + 2y + 1 + z^2 = 2 + \frac{1}{4} + 1$$

$$\left(x+\frac{1}{2}\right)^2 + (y+1)^2 + z^2 = \frac{13}{4}$$

Center $= \left(-\frac{1}{2}, -1, 0\right)$, $r = \frac{\sqrt{13}}{2}$

intersects xz plane when $y=0$

$$\left(x+\frac{1}{2}\right)^2 + 1 + z^2 = \frac{13}{4}$$

$$\left(x+\frac{1}{2}\right)^2 + z^2 = \frac{9}{4}$$

$\boxed{\text{intersects } xz \text{ plane in a circle}}$

15. Given $\mathbf{a} = \langle 1, 5, 7\rangle$, $\mathbf{b} = \langle 2, 0, 5\rangle$, find

(i) $2\mathbf{a} - \frac{1}{2}\mathbf{b}$

(ii) A unit vector in the direction of $\mathbf{b}$

(iii) The cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

(i) $2a - \frac{1}{2}b = 2\langle 1, 5, 7\rangle - \frac{1}{2}\langle 2, 0, 5\rangle$

$= \langle 2, 10, 14\rangle - \langle 1, 0, \frac{5}{2}\rangle$

$= \langle 1, 10, \frac{23}{2}\rangle$

(ii) $u = \frac{b}{|b|} = \frac{\langle 2, 0, 5\rangle}{\sqrt{4+25}} = \left\langle \frac{2}{\sqrt{29}}, 0, \frac{5}{\sqrt{29}}\right\rangle$

(iii) $\cos\theta = \frac{\langle 1, 5, 7\rangle \cdot \langle 2, 0, 5\rangle}{\sqrt{1+25+49}\sqrt{4+25}} = \frac{37}{\sqrt{75}\sqrt{29}}$

$\theta = \arccos\left(\frac{37}{\sqrt{75}\sqrt{29}}\right)$

$\vec{a} \perp \vec{b} \leftrightarrow \vec{a} \cdot \vec{b} = 0$

16. Find the value(s) of x so that the vectors $\langle x, x, -1\rangle$ and $\langle 1, x, 6\rangle$ are orthogonal.

$\langle x, x, -1\rangle \cdot \langle 1, x, 6\rangle = 0$

$x + x^2 - 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\boxed{\begin{array}{l} x = -3 \\ x = 2 \end{array}}$

17. Given $\mathbf{c} = \langle 1, 3, 2\rangle$, $\mathbf{d} = \langle 1, -4, 1\rangle$, find the scalar and vector projection of $\mathbf{c}$ onto $\mathbf{d}$.

scalar projection of $\vec{c}$ onto $\vec{d}$ is

$\text{comp}_{\vec{d}}\vec{c} = \frac{\vec{c} \cdot \vec{d}}{|\vec{d}|} = \frac{\langle 1, 3, 2\rangle \cdot \langle 1, -4, 1\rangle}{\sqrt{18}}$

$= \frac{-9}{\sqrt{18}}$

vector projection of $\vec{c}$ onto $\vec{d}$ is

$\text{proj}_d c = \frac{c \cdot d}{|d|} \frac{d}{|d|}$

$= \frac{-9}{\sqrt{18}} \frac{\langle 1, -4, 1\rangle}{\sqrt{18}}$

$= -\frac{1}{2}\langle 1, -4, 1\rangle$

$= \boxed{\left\langle -\frac{1}{2}, 2, -\frac{1}{2}\right\rangle}$