Staggered level repulsion in lead-symmetric transport

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with: M Kopp, S Rotter

Banff, 28 February 2008
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Overview

• Motivation: Transport in mesoscopic systems
• Symmetric systems
• RMT: staggered level repulsion
• Large number of channels
• Appendix: details of the calculation
Transport in mesoscopic systems

Marcus group
Transport in mesoscopic systems

S matrix \[ S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \]
evals \( T_n \) of \( t^\dagger t \)

- conductance \[ G = \frac{e^2}{h} \sum_{n=1}^{N} T_n \]
- shot noise \[ P = \frac{2e^2}{h} V \sum_{n=1}^{N} T_n (1-T_n) \]
Scattering matrix from circular ensemble (COE: $\beta=1$; CUE: $\beta=2$; CSE: $\beta=4$)

Joint pdf of transmission eigenvalues

$$P\left(\{T_n\}\right) \propto \prod_{n<m} |T_n - T_m|^{\beta} \times \prod_k T_k^{\frac{1}{2}(\beta-2)}$$

- Level repulsion (UCF)
- 1-point density (WL)

(Baranger & Mello 1994; Jalabert, Pichard & Beenakker 1994)
lead-preserving symmetries

- desymmetrization

(Baranger & Mello 1996)

Superposition of transmission eigenvalues

(reduced repulsion)
lead-transposing symmetries

- desymmetrization
- transmission matrix \( t = \frac{1}{2} (S_+ - S_-) \)
  \[
  t^\dagger t = \frac{1}{4} (2 - U - U^\dagger), \quad U = S_+ S_-^\dagger, \quad T_n = \sin^2 \frac{\Theta_n}{2}
  \]
- mixes parities ( \([\text{current}, \text{symmetry}] \neq 0\) )
previous observations

RMT: $U$ from COE

• $\text{var } G$ increases by factor 2  \hspace{1cm} (Baranger & Mello)

• no WL corrections \hspace{1cm} (S Rotter & co: Numerics)

• one-point function \hspace{1cm} (Gopar, Rotter & HS)

$$T_n = \sin^2 \frac{\Theta_n}{2}, \hspace{0.5cm} \Theta_n \text{ uniform } \Rightarrow \rho(T) = \frac{1}{\pi \sqrt{T(1-T)}}$$
here: complete statistics ($\beta=1$)

- $U = S^+_+ S^-_\dagger$ from COE
- $P(\Theta_n) \propto \prod_{n<m} \left| \sin \left( \frac{\Theta_n - \Theta_m}{2} \right) \right|$
- $T_n = \sin^2(\Theta_n / 2)$
- can be realized by

$$\Theta_n = \pm 2 \arcsin \sqrt{T_n}$$
here: complete statistics ($\beta=1$)

- $U = S_+ S_-^\dagger$ from COE

- $P(\Theta_n) \propto \prod_{n<m} \sin\left(\frac{\Theta_n - \Theta_m}{2}\right)$

- $T_n = \sin^2(\Theta_n / 2)$

- combinatorics over $\text{sgn}\,\Theta_n$:
  - order $T_1 \leq T_2 \leq \cdots \leq T_N$
  - pdf as a Vandermonde det.
  - sum over $\text{sgn}\,\Theta_n$

- determinant factorizes

$\text{(odd indices vs even indices)}$

For details see appendix
Final result

\[ P(\{T_n\}) \propto \left( \prod_{m>n}^{\text{both odd}} (T_m - T_n) \right) \cdot \left( \prod_{m>n}^{\text{both even}} (T_m - T_n) \right) \cdot \left( \prod_{l \text{ odd}}^{l+N \text{ even}} \frac{1}{\sqrt{1-T_l}} \right) \]  

Reduced level repulsion  
\[ \rightarrow \] enhanced fluctuations

Symmetric weight
\[ \rho(T) = \frac{1}{\pi \sqrt{T(1-T)}} \]
no 1/N corrections (WL)

\[ T_1 \leq T_2 \leq \cdots \leq T_N \]  
\[ \rightarrow \] Staggered level sequence

(magnitude, not: parity/ill defined)
staggered level sequences (lead-transposing)

uncorrelated level sequences (lead-preserving)
Nearest-neighbour spacing $s = T_{n+1} - T_n$
Test: Model systems

quantum billiards

open kicked rotators

\[ P(s) \]

\[ s \]

(a) asymmetric symmetric

(b) asymmetric symmetric

Diagram showing plots of quantum billiards and open kicked rotators with different symmetric and asymmetric conditions.
\( n \) th-nearest neighbour statistics

- large \( N \): statistics of staggered & independent superpositions converge
large-$N$ asymptotics

Observation: *ignore weights* \[ \left( \prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left( \prod_{l+N \text{ even}} \frac{1}{\sqrt{1-T_l}} \right) \]

uncorrelated superposition (2+2 levels)
\[ (T_2 - T_1)(T_4 - T_3) + (T_3 - T_1)(T_4 - T_2) + (T_4 - T_1)(T_3 - T_2) \]
\[ = 2(T_3 - T_1)(T_4 - T_2) \]

staggered superposition
• holds for all $N$
• large $N$: continuum approx: weights \(\rightarrow\) constant

(low-order) correlation functions all converge to superposition of two uncorrelated level sequences (w/o WL)
Summary

Transport in systems with lead-transposing symmetry:

• Mixes parities
• Joint pdf: staggered levels, no direct repulsion
• For many channels: like uncorrelated level sequences (as if system could be desymmetrized)

• Dynamical mechanism? (semiclassics?)

preprint: arxiv:0708.0690
Appendix: Details of the calculation

- **Order eigenphases**

  \[ |\Theta_1| \leq |\Theta_2| \leq |\Theta_3| \leq \ldots \leq |\Theta_N| \leq \pi \quad \sigma_n = \text{sgn} \Theta_n \]

  \[ P_\Theta(\{\Theta_n\}) \propto \prod_{m>n} \left[ \sigma_m \sin \frac{\Theta_m - \Theta_n}{2} \right] \propto \prod_{l \text{ even}} \sigma_l \prod_{m>n} \sin \frac{\Theta_m - \Theta_n}{2} \]

- **Vandermonde determinant**

  \[ \prod_{m>n} \sin \frac{\Theta_m - \Theta_n}{2} = (-i)^{N(N-1)/2} \det B(\{\sigma_n \theta_n\}) \]

  \[ B_{ml}(\{\Theta_n\}) = \exp(i \Theta_m l), \ m = 1, 2, 3, \ldots, N \]

  \[ l \text{ runs in integer steps from } -(N-1)/2 \text{ to } (N-1)/2 \]
• sum over $\text{sgn} \Theta_n$

$$P_\theta(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_\Theta(\{\sigma_n \theta_n\})$$

multilinearity of the determinant

$$P_\theta(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C,$$

$$C_{ml} = 2 \cos(\theta_m l) \text{ for odd } m$$

$$C_{ml} = 2i \sin(\theta_m l) \text{ for even } m$$
• **sum over** \( \text{sgn} \Theta_n \)

\[
P_\theta(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_\Theta(\{\sigma_n\theta_n\})
\]

multilinearity of the determinant

\[
P_\theta(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C
\]

\[
C_{ml} = 2\cos(\theta_m l) \text{ for odd } m
\]

\[
C_{ml} = 2i\sin(\theta_m l) \text{ for even } m
\]

• for every \( l > 0 \): add \( l \)th column to \(-l\)the column

.erase

\(\rightarrow\) cancels all sine terms in the latter columns
• **sum over** \( \text{sgn} \Theta_n \)

\[
P_{\theta}(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_{\Theta}(\{\sigma_n \theta_n\})
\]

multilinearity of the determinant

\[
P_{\theta}(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C,
\]

\( C_{ml} = 2 \cos(\theta_m l) \) for odd \( m \)

\( C_{ml} = 2i \sin(\theta_m l) \) for even \( m \)

• for every \( l > 0 \): add \( l \)th column to \(-l\)th column

\[\rightarrow\] cancels all sine terms in the latter columns

• **determinant factorises** \( \det C = \det D \det E \)
$N$ odd

\[ \det C = \det D \det E \]

\[ D_{ml} = \cos \theta_m l, \ m \ odd \quad l = 0, 1, 2, \ldots, (N - 1)/2 \text{ for the matrix } D \]

\[ E_{ml} = \sin \theta_m l, \ m \ even \quad l = 1, 2, \ldots, (N - 1)/2 \text{ for the matrix } E \]

$D_{ml}$: polynomial in $\cos \theta_m$ of degree $l$

$E_{ml}$: $\sin \theta_m$ times polynomial of degree $l - 1$

only need to keep highest monomial $\cos^l \theta_m$

\[ \rightarrow \quad \text{Vandermonde determinant} \]

\[ \det D \propto \prod_{m > n, \text{ both odd}} (\cos \theta_n - \cos \theta_m), \]

\[ \det E \propto \prod_{l \text{ even}} \sin \theta_l \prod_{l > n, \text{ both even}} (\cos \theta_n - \cos \theta_m) \]
$N$ even

$$\det C = \det D \det E$$

$$D_{ml} = \cos \theta_m l, \; m \text{ odd}$$

$$E_{ml} = \sin \theta_m l, \; m \text{ even}$$

$$l = 1/2, 3/2, \ldots, (N - 1)/2$$

$D_{ml}$: $\cos(\theta_m/2)^l$ times polynomial in $\cos \theta_m$ of degree $(l - 1)/2$

$E_{ml}$: $\sin(\theta_m/2)^l$

only need to keep highest monomial $\cos^l \theta_m$

$\rightarrow$ Vandermonde determinant

$$\det D \propto \prod_{l \text{ odd}} \cos(\theta_l/2) \prod_{m > n, \text{ both odd}} (\cos \theta_n - \cos \theta_m)$$

$$\det E \propto \prod_{l \text{ even}} \sin(\theta_l/2) \prod_{m > n, \text{ both even}} (\cos \theta_n - \cos \theta_m)$$
transformation to $T_n = \sin^2 \frac{\Theta_n}{2}$

$$\cos \theta_n - \cos \theta_m = 2(T_m - T_n)$$

$$\sin(\theta_l/2) = \sqrt{T_l}$$

$$\cos(\theta_l/2) = \sqrt{1 - T_l}$$

$$\sin(\theta_l) = 2\sqrt{T_l(1 - T_l)}.$$ 

Jacobian $d\theta_n/dT_n = 1/\sqrt{T_n(1 - T_n)}$
Final result

\[
P(\{T_n\}) \propto \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both odd}} \cdot \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both even}} \cdot \left( \prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left( \prod_{l \text{ even}} \frac{1}{\sqrt{1-T_l}} \right)
\]

\[
P(\{T_n\}) \propto \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both odd}} \cdot \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both even}} \cdot \left( \prod_{l \text{ odd}} \frac{1}{\sqrt{T_l(1-T_l)}} \right)
\]

or:

\[
P(\{T_n\}) \propto \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both odd}} \cdot \left( \prod_{m>n} (T_m - T_n) \right)^{\text{both even}} \cdot \left( \prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left( \prod_{l+N \text{ even}} \frac{1}{\sqrt{1-T_l}} \right)
\]