Introduction – Ehrenfest time in scattering problems

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Collaborations & Discussions:  P. Jacquod, C. Petitjean, P. Brouwer, E. Sukhorukov
When is quantum = classical?

**QUANTUM**
- Ehrenfest theorem
  \[ \frac{d}{dt} \hat{\rho} = -i\hbar [\hat{H}, \hat{\rho}] \]
- Commutator
- Phase-space
- Wigner function

**CLASSICAL**
- Liouville theorem
  \[ \frac{d}{dt} f(r, p) = -\{\hat{H}, f(r, p)\} \]

Moyal bracket ≠ Poisson bracket

Minimally dispersed wavepacket
\[ L^{1/2} \lambda^{1/2}_{\text{deBoglie}} \]

Hyperbolic stretch
Quantum = Classical

Non-hyperbolic stretch & fold
Quantum ≠ Classical
Ehrenfest time (log time)

Ehrenfest Time, $\tau_E$:
time during which
quantum = classical

$$\tau_E = 2\Lambda^{-1} \ln[\mathcal{L}/\lambda_{\text{de Broglie}}]$$
with $\mathcal{L} = L, (W^2/L)$

WAVEPACKET growth $\propto \exp[-\Lambda t]$

TIMESCALES:

- ♠ (Level-spacing)$^{-1} \propto (\mathcal{L}/\lambda_{\text{de Broglie}})$
- ♠ Ehrenfest time $\propto \ln[\mathcal{L}/\lambda_{\text{de Broglie}}]$
- ♠ Classical scales:
dwell time = $\lambda_{\text{de Broglie}}$-independent

CLOSED SYSTEM – $\sim$

OPEN SYSTEM – classical limit

"All" particles escape before Ehrenfest time

Form $\mathcal{F}$

- $\sim$AL’ Brouwer–Rahav–Tian
A short and inaccurate history of the Ehrenfest time!

LAST CENTURY

Ehrenfest: Ehrenfest theorem. \( \neq \) Liouville theorem
Z. Physik 45 455 (1927)

??? Larkin-Ovchinnikov: superconducting fluct. in metals

Berman-Zaslavsky: stochasticity

Aleiner-Larkin weak-loc. in transport & level-statistics
PRB 54, 14423 (1996); 55, R1243 (1997).

THIS CENTURY  Semiclassics  
... see other talks this week.

♣ Cut-off on loops  Sieber, Müller-Braun-Heusler-Haake

♣ Off-diagonal (Noise, weak-loc., UCFs, etc) suppressed (or not) Rahav-Brouwer & Jacquod-Whitney

Weak localization and back-scattering

Conductance fluctuations

Fano factor of shot noise
Open systems – scattering matrix

Scattering matrix, \( S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix} \)

Landauer-Büttiker: eigenvalues of \( t^\dagger t \rightarrow \) all transport properties

- ♣ Electrical conductance \( \propto \sum_n T_n \)
- ♣ quantum noise \( \propto \sum_n T_n (1 - T_n) \)

Quantum noise : measures determinism
Bands in classical phase-space

Whitney-Jacquod (2005)

Open systems:
Paths in *families* (bands)

cf. integrable systems

Paths associated with 4 band corners
dynamics relative to black path
Phase-space basis: orthogonal basis of wavepackets

Cover: bands with area $> 2\pi \hbar$

$\Rightarrow$ paths with $t < \tau_E$

$E \lambda \sim 1/2$

de Broglie

Fourier transform unchanged under

Basis states

CLASSICAL block: diagonal

$| \text{eigenvalues} | = 0, 1$

QUANTUM block: not diagonal

semclassics shows non-RMT.

Eigenvalues of $t^t t = 0, 1$ $\Rightarrow T_n(1-T_n)=0$ $\Rightarrow$ Noise-less Modes
Conclusion

Open systems: Ehrenfest time controls cross-over to NEW universal regime (non-RMT) as $\lambda_{\text{deBroglie}} \to 0$.

Open questions

[A] Ehrenfest time relevant for closed systems?
- need classical timescale; i.e. inter-billiard transfer time

[B] Re-summing loop expansion for finite Ehrenfest time?

[C] Negative Ehrenfest time $\tau_E = \Lambda^{-1} \ln[\mathcal{L}/\lambda_{\text{deBroglie}}]$ ??
... for noise $\mathcal{L} = W^2/L \to 0$ for $W \ll L$.

[D] Ehrenfest time in graphs?
- require correlated actions?
Typical Ehrenfest times

QUANTUM

\[ \mathcal{L} \sim \lambda_{\text{deBroglie}} \]

\[ \tau_E = \Lambda^{-1} \ln[1] \sim 0 \]


Quantum dot

CLASSICAL

\[ \mathcal{L} \sim 10^3 \lambda_{\text{deBroglie}} \]

\[ \tau_E = \Lambda^{-1} \ln[10^3] \sim 7\tau_0 \]

Double pendulum

www.chaoticpendulums.com

\[ \mathcal{L} \sim 10^{34} \lambda_{\text{deBroglie}} \]

\[ \tau_E = \Lambda^{-1} \ln[10^{34}] \sim 80\tau_0 \]

Schrodinger cat state after 2 minutes !!

Decoherence??

Zurek (2003)