# Calculus

Instructions   Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Suppose that w denotes the composite function $u \circ v$ [in other words, $w(x) = u(v(x))$], and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Use the chain rule to find the value of the derivative $w'(0)$. [This is exercise 60 on page 193 of the textbook.]

   Solution. According to the chain rule, $w'(x) = u'(v(x))v'(x)$. Substituting 0 for x shows that $w'(0) = u'(v(0))v'(0) = u'(2)v'(0) = 4 \times 5 = 20$.

2. In the figure, the three curves represent the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Which curve is which? How do you know?
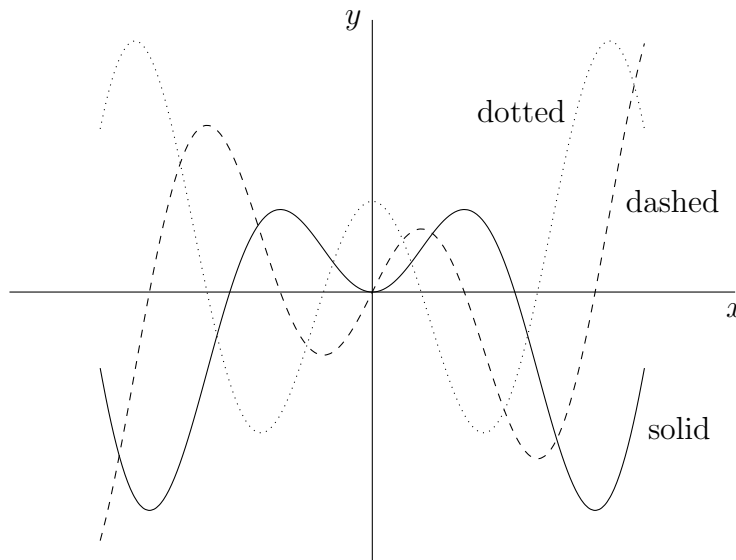
   Solution. When x is small and positive, the dotted curve is decreasing, but the other two curves both have positive values, so they cannot represent the derivative of the function corresponding the dotted curve. If the problem is well posed, then the dotted curve must represent the graph of f''.

Calculus

The solid curve does not cross the x-axis when the dashed curve has a horizontal tangent line, so the solid curve cannot represent the derivative of the dashed curve. Consequently, the solid curve must correspond to the function f, and the dashed curve must correspond to the derivative f'.

Now you can confirm the above deduction by observing that the dashed curve is above the x-axis when the solid curve is increasing and below the x-axis when the solid curve is decreasing. Similarly, the dotted curve is above the x-axis when the dashed curve is increasing and below the x-axis when the dashed curve is decreasing.

3. Jules is working on exercise 16 on page 198 of the textbook: find dy/dx by implicit differentiation for the curve given by the equation

$$x \cos y + y \cos x = 1.$$

"When I type the command `impDiff(x*cos(y)+y*cos(x)=1,x,y)` in my TI-89 calculator," says Jules, "I get $\dfrac{y \sin(x) - \cos(y)}{\cos(x) - x \sin(y)}$ as the answer, but Maple gives the different answer $-\dfrac{x \sin(y) - \cos(x)}{- \cos(y) + y \sin(x)}$ when I type the command `implicitdiff(x*cos(y)+y*cos(x)=1,x,y)`. I don't know which answer is right."

Help Jules out by computing dy/dx by hand in this example.

Solution. Differentiating the given equation using the product rule and the chain rule (assuming that y represents some function of x) gives the equation

$$-x(\sin y)y' + \cos y - y \sin x + y' \cos x = 0,$$
$$\text{or} \qquad y'(-x \sin y + \cos x) = -\cos y + y \sin x.$$

Dividing gives the result

$$\frac{dy}{dx} = \frac{-\cos y + y \sin x}{-x \sin y + \cos x},$$

which agrees with the answer from the TI-89 calculator. Maple uses a different syntax: the command Jules typed into Maple computes the derivative dx/dy instead of dy/dx.