Linear Algebra

1. Find the vector projection of the vector $\begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix}$ onto the vector $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

   [This is exercise 3d on page 224.]

   Solution. The projection of $\mathbf{x}$ onto $\mathbf{y}$ equals $\left(\mathbf{x}^T \frac{\mathbf{y}}{\|\mathbf{y}\|}\right) \frac{\mathbf{y}}{\|y\|}$. In our case, $\mathbf{x}^T\mathbf{y} = 2 - 10 - 4 = -12$, and $\|\mathbf{y}\|^2 = 1^2 + 2^2 + (-1)^2 = 6$, so the answer is

$$\frac{-12}{6}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 2 \end{pmatrix}.$$

2. Find the distance from the point in R^3 with coordinates $(1, 1, 1)$ to the plane defined by the equation $2x + 2y + z = 0$. [exercise 9, p. 224]

   Solution. The vector $(1, 1, 1)^T$ joins a point on the plane (namely, the point $(0, 0, 0)$) to the given point. The indicated distance is the length of the projection of this vector onto the direction normal to the plane. From the equation of the plane, one reads off the normal vector $(2, 2, 1)^T$. The unit normal vector is $\frac{(2,2,1)^T}{\sqrt{2^2+2^2+1^2}}$, so the final answer is

$$\frac{2+2+1}{\sqrt{9}} = \frac{5}{3}.$$