Linear Algebra

Write your **name**: ______________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points each)

1. If A is a 12×5 matrix (that is, A has 12 rows and 5 columns), then the null space of A has dimension at least 7. `True` `False`

2. The function $L : R^2 \to R^1$ defined by $L(\mathbf{x}) = \|\mathbf{x}\|$ (that is, the norm of $\mathbf{x}$) is a linear transformation. `True` `False`

3. The matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ is similar to the matrix $\begin{pmatrix} 2 & 4 \\ 0 & 6 \end{pmatrix}$. `True` `False`

4. If A is a 3×3 matrix of rank 2, then the dimension of the null space of A^T (the transpose) is equal to 2. `True` `False`

5. If a 2×2 matrix of real numbers has purely imaginary eigenvalues, then the determinant of the matrix is negative. `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. If L is the linear operator on R^2 that doubles the length of each vector and also rotates each vector by 30° counterclockwise, then the standard matrix representation of L is $\begin{pmatrix} \sqrt{3} & \square \\ \square & \square \end{pmatrix}$.

7. If the scalar product of two vectors in R^3 is equal to 0, then the two vectors are said to be ______________ .

8. When $b = \square$, the linear system $\left\{ \begin{matrix} 1x_1 + 2x_2 = 5 \\ 2x_1 + bx_2 = 0 \end{matrix} \right\}$ has $x_1 = -1$ and $x_2 = 1$ as a solution in the sense of least squares.

9. Suppose a linear transformation $L : R^2 \to R^2$ has the standard matrix representation $\begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$. If $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, then the matrix representation of L with respect to the basis $[\mathbf{u}_1, \mathbf{u}_2]$ is $\begin{pmatrix} \square & 0 \\ \square & \square \end{pmatrix}$.

Linear Algebra

In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points each)

10. Suppose $A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 4 & -4 & 5 & -5 \end{pmatrix}$. Find an orthonormal basis for the null space of the matrix A.

Linear Algebra

11. Suppose $A = \begin{pmatrix} 7 & 1 & -4 \\ 4 & 4 & -4 \\ 0 & 0 & 0 \end{pmatrix}$. Find a diagonal matrix that is similar to A.

Linear Algebra

12. Consider the inner product space of continuous functions on the interval $[-1, 1]$, where $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\,dx$. Find the projection of the function x^2 onto the subspace spanned by the two functions 1 and x.