

Linear Algebra

Instructions Please use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. Consider the system

$$\begin{cases} 2x_1 + x_2 = a^2 \\ 6x_1 + 3x_2 = a \end{cases}$$

of simultaneous equations for the unknowns x_1 and x_2 , where a is a certain constant. For which value(s) of the constant a is the system of equations *consistent*? How do you know?

Linear Algebra

2. Rose is studying the linear system

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\5x_1 + 6x_2 + 7x_3 &= 8 \\9x_1 + 10x_2 + 11x_3 &= 12\end{aligned}\tag{†}$$

of three equations in the three unknowns x_1 , x_2 , and x_3 . Rose discovers that the TI-89 calculator has a command `rref` (which stands for “reduced row echelon form”), and the command

`rref([1,2,3,4;5,6,7,8;9,10,11,12])`

returns the output

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

What should Rose conclude about the set of solutions of the linear system (†)?