

# Linear Algebra

**Instructions** Please answer the five problems on your own paper. These are essay questions: you should write in complete sentences.

1. Jordan is using a TI-89 calculator to help analyze the linear system  $A\mathbf{x} = \mathbf{b}$ , where  $A$  is a certain  $3 \times 4$  matrix and  $\mathbf{b}$  is a certain  $3 \times 1$  matrix (a column vector). Jordan applies the `rref` command to the augmented coefficient matrix and obtains the result

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(Jordan's calculator does not show a vertical bar to separate the last column from the coefficient matrix.) Discuss what information Jordan can deduce about the original linear system.

[For instance, is the system underdetermined or overdetermined? consistent or inconsistent? Is there a unique solution? Does Jordan have enough information to write down the solution(s)?]

2. Consider the system of three simultaneous equations

$$\begin{cases} x_1 + x_2 & = 2 \\ ax_1 + ax_2 & = 3a \\ bx_1 + bx_2 + ax_3 & = 4 + b \end{cases}$$

for the unknowns  $x_1$ ,  $x_2$ , and  $x_3$ . Find all values of  $a$  and  $b$  for which this system of equations is consistent.

Explain your reasoning in complete sentences.

3. Suppose

$$A = \begin{bmatrix} 0 & a & 1 \\ 1 & 0 & 1 \\ 0 & 0 & a \end{bmatrix}.$$

Determine the value(s) of  $a$  for which the matrix  $A$  is invertible.

[If you do a computation to solve this problem, say what computation you are doing and why.]

# Linear Algebra

4. Suppose that  $A$  is an  $n \times n$  matrix, and  $S$  is an invertible  $n \times n$  matrix. Show that  $\det(S^{-1}AS) = \det(A)$ .
  
5. Maude is studying the set of all polynomials in  $x$  of odd degree. Help Maude decide if this *set* forms a *vector space* (under the usual operations of addition and scalar multiplication).