Complex Variables

Instructions   Please write your solutions on your own paper. Explain your reasoning in complete sentences to maximize credit.

1. Either sketch or describe in words the set of non-zero complex numbers z for which $\operatorname{Re}(i/z) = 0$.

2. Determine (in the standard form $x + iy$) all complex numbers z such that $z^4 + 4 = 0$.

3. Explain how you know that the "punctured plane" consisting of all non-zero complex numbers is both an open set and a connected set.

4. Does the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2 - i^n}$ converge or diverge? Explain how you know.

5. Find all (infinitely many) values of the expression $i^{(2+i)}$. Write the values in the standard form $a + bi$.

6. Evaluate the line integral $\displaystyle\int_{\gamma} \frac{1}{z}\, dz$, where γ is a quarter circle in the first quadrant joining the point 1 to the point i. Write the answer in the standard form $a + bi$.

7. Suppose $f(z) = |z|^2$. Show that the (complex) derivative $f'(z)$ does not exist unless $z = 0$.

8. Determine the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n} z^{2n}$.