Complex Variables

Instructions   Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Determine the residue of the function $\dfrac{4z^2}{z^9 - 1}$ at the simple pole where $z = 1$.

2. The function $\dfrac{1}{z(1 - z)}$ is analytic in the punctured unit disc (where $0 < |z| < 1$). Determine the Laurent series for this function (in powers of z and $1/z$) that converges in this punctured disc.

Complex Variables

3. Evaluate the complex line integral

$$\int_{|z|=1} \frac{\cos(z)}{\sin(z)}\, dz,$$

where the integration path is the unit circle oriented in the standard counterclockwise direction.