

Complex Variables

Instructions Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Find a linear fractional transformation that takes the triple of points $(0, 1, 2)$ onto $(1, 2, 3)$.

Solution. This transformation is a translation by 1 unit to the right, so you can write down the formula without any calculation: the transformation is $z \mapsto z + 1$.

2. Find a linear fractional transformation that takes the triple $(1, 0, \infty)$ onto $(\infty, 1, 0)$.

Solution. Suppose the transformation has the general form $\frac{az + b}{cz + d}$. Since 1 maps to ∞ , we must have $c + d = 0$, or $c = -d$. Since 0 maps to 1, we have $b/d = 1$, or $b = d$. Since ∞ maps to 0, we have $a/c = 0$, or $a = 0$. We may take d equal to 1, in which case $b = 1$ and $c = -1$; the transformation is given by $\frac{1}{-z + 1}$.

3. Find a linear fractional transformation that takes the circle centered at 0 with radius 1 onto the circle centered at 3 with radius 2.

Solution. You can implement this transformation by first dilating by a factor of 2 and then translating 3 units to the right. Thus the transformation is $z \mapsto 2z + 3$.