Examination 1

Instructions: Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Which of the two complex numbers $(1-i)^{40}$ and $(1-i)^7$ has the larger imaginary part? Explain how you know.

2. Determine the set of values of the complex variable z for which

$$\operatorname{Re}\left(\frac{2}{z}\right) > 1,$$

and sketch a picture representing this set.

3. Determine the three values of the complex variable z for which $z^3 = i$. Write each value in Cartesian form as $a + bi$.

4. Find all values of the complex variable z for which

$$(\log z)^2 = 1$$

(all possible branches of the logarithm). Explain your reasoning.

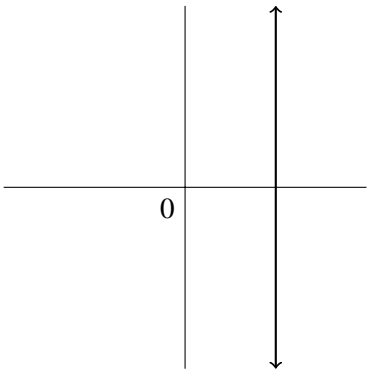
5. Does complex conjugation commute with taking the exponential? In other words, is it correct to say that

$$\exp(\overline{z}) = \overline{\exp(z)} \ ?$$

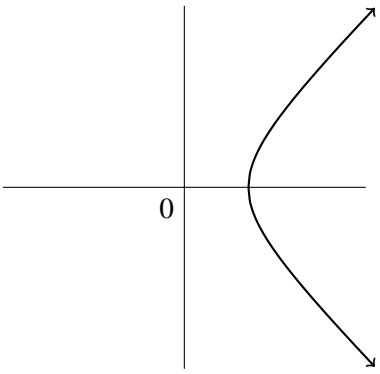
Explain why or why not.

6. Is it correct to say that $\left(2^2\right)^i = 2^{2i}$? Explain why or why not.

Extra Credit Problem. If $w = f(z)$, and the curves with arrows shown in the pictures below correspond with respect to this function, could $f(z)$ be equal to z^2 or e^z or $\sin(z)$ or none of these? Explain how you know.



z plane



w plane