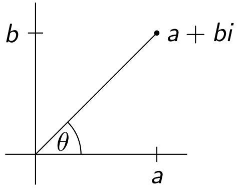
Notation that you read about

- *Real part* of $a + bi$, written $\mathrm{Re}(a + bi)$, equals a.
- *Imaginary part* of $a + bi$, written $\mathrm{Im}(a + bi)$, equals b (NOT bi).
- *Conjugate* of $a + bi$, written $\overline{a + bi}$, equals $a - bi$. (Reflection of the vector in the horizontal axis.)
- *Modulus* or *absolute value* of $a + bi$, written $|a + bi|$, equals $\sqrt{a^2 + b^2}$.
- *Argument* of $a + bi$, written $\arg(a + bi)$, equals the polar angle θ.



What should $\cos(2+3i)$ mean?

Some formulas from real calculus:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$
$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

The right-hand sides make sense when the real number x is replaced by a complex number z, so the series can be used to define e^z and $\sin(z)$ and $\cos(z)$ when z is a complex number.

A magic formula

Using series expansions:

$$\begin{aligned} e^{iz} &= 1 + iz + \frac{(iz)^2}{2!} + \frac{(iz)^3}{3!} + \frac{(iz)^4}{4!} + \cdots \\ &= 1 + iz - \frac{z^2}{2!} - i\frac{z^3}{3!} + \frac{z^4}{4!} + \cdots \\ &= \cos(z) + i\sin(z) \end{aligned}$$

(known as Euler's formula).

Example

$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1.$

Quiz due at the beginning of next class

1. If z denotes the complex number $2 - i$, then list the following real numbers in increasing order: $\mathrm{Im}(z)$, $|z|$, $\mathrm{Re}(z^2)$.
2. If z denotes the complex number $1 + i$, then compute $z + \frac{1}{z}$ and express the answer in the form $a + bi$.
3. Find all values of the complex variable z for which $|z - 1| = |z + 1|$.