The complex logarithm

If the logarithm and the exponential function are to be inverses, then

$$e^{\log(z)} = z = re^{i\theta} = e^{\ln(r)+i\theta}$$

so the only possible definition for the complex logarithm is

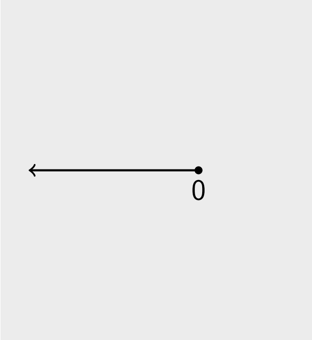
$$\log(z) = \ln|z| + i\arg(z).$$

Problem: There are infinitely many values for $\arg(z)$.

Solution: We can define $\arg(z)$ uniquely on the plane with a cut that prevents us from circling the origin.

Principal branch of the logarithm

For this branch, make a cut along the negative part of the real axis.



principal value:
$-\pi < \theta < \pi$

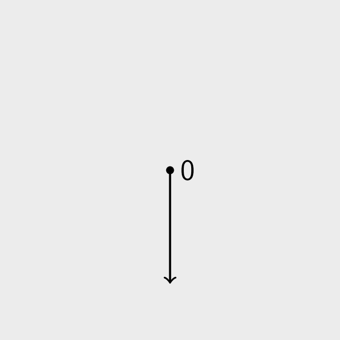
For the principal branch,

$$\log(-1 - i) = \ln\sqrt{2} - \frac{3\pi i}{4}.$$

The textbook writes Log (capital letter) for the principal branch.

A nonstandard branch of the logarithm

For this branch, make a cut along the negative part of the imaginary axis.



$-\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

For this nonstandard branch,

$$\log(-1 - i) = \ln \sqrt{2} + \frac{5\pi i}{4}.$$

Assignment due next class

- Section I.6: Exercise 2(a),(b)
- Section I.4: Exercise 1(b),(c)
- Section I.1: Exercise 1(i)

Quiz

1. Rewrite $e^{407\pi i}$ in the form $a + bi$.
2. Draw a picture of the set $\{\, z \in \mathbb{C} : 0 < \mathrm{Re}(z) < 1 \,\}$.
3. Find all three values of $(-8)^{1/3}$.