

Reminder on the complex logarithm

$$\log(z) = \ln |z| + i \arg(z)$$

General powers

When z and w are complex numbers, what should z^w mean?

If \exp and \log are to be inverses, then

$$z^w = \exp(\log(z^w)) = \exp(w \log z),$$

so it is reasonable to *define* z^w to mean $e^{w \log(z)}$.

Example

$$\begin{aligned}i^{1+i} &= e^{(1+i) \log(i)} \\&= e^{(1+i)i(\frac{\pi}{2}+2\pi n)} \\&= e^{-\frac{\pi}{2}+2\pi n} e^{i(\frac{\pi}{2}+2\pi n)} \\&= ie^{-\frac{\pi}{2}+2\pi n}.\end{aligned}$$

Assignment due next class

- ▶ Section 1.7: Exercise 1(a),(b)