Reminder

Exam 2 takes place on Thursday, October 26.

The material covered is Sections 2–7 of Chapter II.

Geometric interpretation of complex-linear transformations

- translation: $z \to z + b$ (where $b \in \mathbb{C}$)
- rotation: $z \to ze^{it}$ (where $t \in \mathbb{R}$)
- dilation: $z \to Rz$ (where $R > 0$)
- a link to an online visualization of transformations by Tim Brzezinski

Composing these functions generates a *group* of transformations, $z \to az + b$, where a and b are complex numbers (and $a \neq 0$).

Linear approximation of analytic functions

$f(z) - f(0) \approx f'(0)z$ when z is close to 0.

[More generally, $f(z) - f(z_0) \approx f'(z_0)(z - z_0)$ when z is close to z_0.]

What does the transformation $z \mapsto f'(0)z$ do geometrically?
If $f'(0) = re^{i\theta}$, then the transformation stretches by a factor of r and rotates by angle θ.

Deduction: If two curves in the z plane cross at 0 at a certain angle, and $w = f(z)$, then the image curves in the w plane cross at $f(0)$ at the same angle.

This deduction depends on the derivative being nonzero; otherwise the angle θ is not well defined.

Analytic functions with *nonzero derivative* are called *conformal mappings*: the angles at which curves cross are preserved.
Here is a link to a visualization tool for conformal mappings by Juan Carlos Ponce Campuzano.

The group of fractional linear transformations

(also called “linear fractional transformations”
or “Möbius transformations”)

Composing transformations of the form $z \mapsto az + b$ with the *inversion* $z \mapsto 1/z$ produces transformations of the form

$$z \mapsto \frac{az+b}{cz+d},$$

where a, b, c, and d are complex numbers.

The derivative equals $\dfrac{ad-bc}{(cz+d)^2}$, so the restriction $ad - bc \neq 0$ is imposed to ensure invertibility of the transformation.