Quiz, September 14

1. Rewrite $e^{407\pi i}$ in the form $a + bi$.

   Solution. One way to solve the problem is to say that

$$e^{407\pi i} = e^{203(2\pi i)+\pi i} = e^{\pi i} = -1.$$

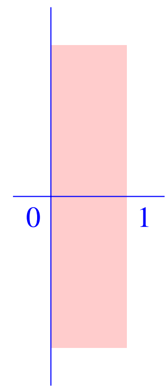
   Alternatively, you could directly apply Euler's formula to say that

$$e^{407\pi i} = \cos(407\pi) + i\sin(407\pi) = -1 + 0i = -1.$$

2. Draw a picture of the set $\{\, z \in \mathbb{C} : 0 < \operatorname{Re}(z) < 1 \,\}$.

   Solution.

   If you write z as $x + yi$, then the set represents the points (x, y) in the plane for which y is unrestricted, and $0 < x < 1$. This set is a vertical strip, as indicated in the figure.

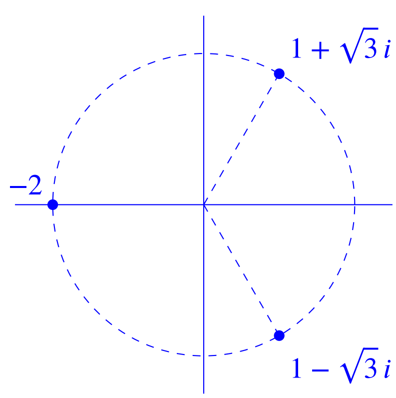


3. Find all three values of $(-8)^{1/3}$.

   Solution. This problem is Exercise 1(e) from Section I.2.

   Write -8 in polar form as $8e^{\pi i}$. One value of $(-8)^{1/3}$ is therefore $8^{1/3}e^{\pi i/3}$, or $2e^{\pi i/3}$, or $2 \cdot (\frac{1}{2} + \frac{\sqrt{3}}{2} i)$, or $1 + \sqrt{3}\, i$.

   Where do the other two values come from? In view of the periodicity of the complex exponential function, the expression $8e^{\pi i}$ is equal to both $8e^{3\pi i}$ and $8e^{5\pi i}$, so the other values of $(-8)^{1/3}$ are $2e^{3\pi i/3}$ and $2e^{5\pi i/3}$. These values simplify to -2 and to $1 - \sqrt{3}\, i$.



   Notice the symmetry in the picture.