Fall 2004 MATH 171

Week in Review XI

courtesy of David J. Manuel

Section 6.1, 6.2, 6.3

Section 6.1

1. Prove $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2. Prove $\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

3. Show by counter-example that $\sum_{i=1}^{n} a_i\, b_i \neq \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right)$

4. Compute $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 - \left(1 + \frac{2i}{n}\right)\right]$

Section 6.2-6.3

5. State the precise definition of $\int_a^b f(x)\,dx$. Define all terms used in this definition.

6. Prove: if f and g are integrable, then $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$.

7. Prove $\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$.

8. Use equally-spaced partitions to write the limit of the sum in 4) as a definite integral.

9. Compute $\int_0^3 \sqrt{9 - x^2}\,dx$.