Fall 2004 MATH 171

Week in Review VI

courtesy of David J. Manuel

Section 3.7, 3.8, and 3.9

Section 3.7

1. Prove the "vector rule" for derivatives: If $\mathbf{r}(t) =< x(t), y(t) >$, then $\mathbf{r}'(t) =< x'(t), y'(t) >$.

2. Prove that $\frac{d}{dt}(\mathbf{r}(t) \cdot \mathbf{s}(t)) = \mathbf{r}(t) \cdot \mathbf{s}'(t) + \mathbf{r}'(t) \cdot \mathbf{s}(t)$.

3. Given $\mathbf{r}(t) =< \cos t, \sin t >$, compute $\mathbf{r}(t) \cdot \mathbf{r}'(t)$. What theorem does your result prove?

Section 3.8

4. Find a formula for the nth derivative of $f(x) = \frac{1}{1-x}$.

5. Use induction (See Appendix E) to prove the formula you found in #4.

6. Find a formula for the second derivative of the product $f(x)g(x)$.

Section 3.9

7. Given $\mathbf{r}(t) =< f(t) \cos t, f(t) \sin t >$ and a value t_0 such that $f'(t_0) = 0$ but $f(t_0) \neq 0$, show that the line tangent to $\mathbf{r}(t)$ at $t = t_0$ is perpendicular to the line from the origin to the point corresponding to $\mathbf{r}(t_0)$.

8. For the parametrized curves below, $x'(t) = y'(t) = 0$ when $t = 0$. Determine the slope of the tangent line by computing $\lim_{t \to 0} \frac{dy}{dx}$.

a) $x = t^3 - 1$, $y = 2t^3$

b) $x = t^5$, $y = t^3$