Fall 2004 MATH 171

Week in Review VII

courtesy of David J. Manuel

Section 3.11, 4.1, 4.2, 4.3

Section 3.11

1. Prove $d(uv) = u\,dv + v\,du$.

2. Define the linearization L of a function f at $x = a$; prove that $L(a) = f(a)$ and $L'(a) = f'(a)$.

Section 4.1

3. Show that the line tangent to $y = e^{rx}$ at $x = a$ has an x-intercept at $x = a - \frac{1}{r}$.

4. Define the *hyperbolic sine* and *hyperbolic cosine* functions as follows:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \ \sinh(x) = \frac{e^x - e^{-x}}{2}$$

a) Prove that $\cosh^2(x) - \sinh^2(x) = 1$

b) Prove that $\frac{d}{dx}(\cosh(x)) = \sinh(x)$.

Section 4.2

5. Prove that the function $f(x) = \frac{2 - 5x}{3 + x}$ is one-to-one and find f^{-1}

6. Given f is differentiable, f' is never 0 and $g = f^{-1}$, assuming g is differentiable, find $g'(a)$.

Section 4.3

7. Prove $\log_a(xy) = \log_a x + \log_a y$.

8. Prove $\lim_{x \to -\infty} e^x = 0$ using the epsilon-delta definition of the limit.

9. Prove $a^x = e^{x \ln a}$ for all x and use this to compute $\frac{d}{dx}(a^x)$.