Fall 2004 MATH 171

Week in Review VIII

courtesy of David J. Manuel

Section 4.4, 4.6, and 4.8

Section 4.4

1. Prove $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

2. Use logarithmic differentiation to differentiate $f(x)g(x)$ and show you obtain the product rule.

3. Prove that, if n is any real number, $\dfrac{d}{dx}(x^n) = nx^{n-1}$ (i.e., prove the power rule for any real exponent).

Section 4.6

4. Define the function $y = \cos^{-1} x$. Compute $\cos^{-1}\left(\cos\left(\dfrac{13\pi}{12}\right)\right)$.

5. Prove $\dfrac{d}{dx}(\arctan\ x) = \dfrac{1}{1+x^2}$

6. Show that $\sin^{-1} x + \cos^{-1} x$ is constant for all x. Find the constant.

Section 4.8

7. If $f(a) = g(a) = 0$, $g'(a) \neq 0$, and f' and g' are continuous at $x = a$, prove that $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to a} \dfrac{f'(x)}{g'(x)}$

8. Prove $\lim\limits_{x\to\infty}\left(1 + \dfrac{1}{x}\right)^x = e$.

9. Prove that, if $p > 0$, then $\lim\limits_{x\to\infty} \dfrac{\ln\ x}{x^p} = 0$. Explain what this result says about the graphs of $f(x) = \ln\ x$ and $g(x) = x^p$.