Fall 2004 MATH 171

Week in Review IX

courtesy of David J. Manuel

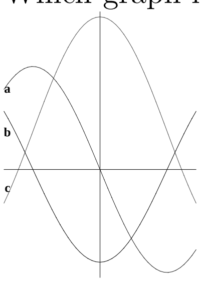
Section 5.1, 5.2, and 5.3

Section 5.1

1. TRUE or FALSE? If f is increasing on (a, b) and increasing on (b, c), then f is increasing on (a, c). If true, prove it. If false, provide a counter-example.

2. Sketch 3 different-shaped graphs which are decreasing on $(-1, 2)$. Explain what is different about each graph.

3. The graphs of a function, its first derivative, and its second derivative are shown below. Which graph is which? Explain your reasoning.



Section 5.2

4. Prove Fermat's Theorem: If f has a local maximum at $x = c$ and $f'(c)$ exists, then $f'(c) = 0$.

5. Sketch the graph of a function f on $[2, 5]$ such that:

a) f has an absolute minimum but no local minimum.

b) f has a local minimum but no absolute minimum.

If either of these is not possible, explain why.

Section 5.3

6. Use the Mean Value Theorem to prove that, if $f' < 0$ on (a, b), then f is decreasing on (a, b).

7. In Section 2.5, you used the Intermediate Value Theorem to prove that an equation such as $x^3 + 3x - 7 = 0$ has a solution. Use the Mean Value Theorem to prove that this equation $(x^3 + 3x - 7 = 0)$ has ONLY one solution.

8. Show that, if two functions have the same derivative, then they differ at most by a constant. i.e., if $f'(x) = g'(x)$ for all x, then there is a constant C such that $f(x) = g(x) + C$.