1. State the precise definition for each of the following statements:
   
   a) \( \lim_{n \to \infty} a_n = L \)
   
   b) \( \lim_{n \to \infty} a_n = \infty \)

2. Use the definition to prove \( \lim_{n \to \infty} \frac{3}{n + 3} = 0 \)

3. Use the definition to prove \( \lim_{n \to \infty} 3^{-n} = 0 \).

4. Prove the following: If \( a_n \to L_1 \) and \( b_n \to L_2 \), then \( a_n + b_n \to L_1 + L_2 \).

5. State and prove the Squeeze Theorem for sequences.

6. State the Monotone Sequence Theorem. Use it to prove that \( \lim_{n \to \infty} \frac{\ln n}{n} \) exists. Find the limit.

7. Use induction to prove that the sequence defined recursively by \( a_0 = 1 \), \( a_{n+1} = \frac{1}{2}a_n + 1 \) is increasing and \( 1 \leq a_n < 3 \) for all \( n \). Explain why the limit exists, then find it.

8. Use induction to prove that the sequence defined in \#7 can be written as \( a_n = \frac{2^{n+1} - 1}{2^n} \) for all \( n \).