Section 4.1 Local Maxima and Minima

Local Extremum: Local Maximum and Local Minimum
We say that $f(c)$ is a **local maximum** if $f(c) \geq f(x)$ for all x near c.
We say that $f(c)$ is a **local minimum** if $f(x) \geq f(c)$ for all x near c.

First Derivative Test for Finding Local Extremum

1. If $f'(x)$ changes from **positive to negative** at c, then $f(c)$ is a **local maximum**.

2. If $f'(x)$ changes from **negative to positive** at c, then $f(c)$ is a **local minimum**.

3. If $f'(x)$ does not change sign at $x = c$, then $f(c)$ is **not** a local extremum.

Definition of a Critical Point

A point $x = c$ is a **critical point** if
1. c is in the domain of the function $f(x)$, and
2. $f'(c) = 0$ or $f'(c)$ does not exist

NOTE!!!

Second Derivative Test for Finding Local Extremum

If p is a critical point of a continuous function f, and $f'(p) = 0$, then

1. If f is concave up at p, then f has a local minimum at p.

2. If f is concave down at p, then f has a local maximum at p.

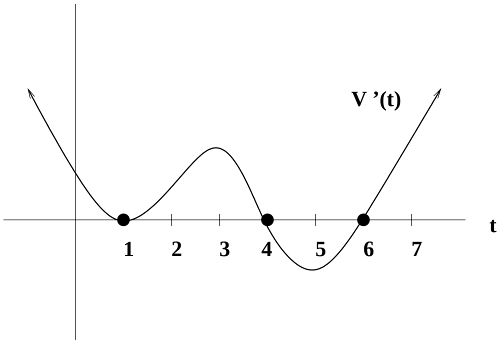
Example: For $f(x) = x^3 - 6x^2 + 9x$, find
(a) all critical values

(b) the open intervals on which f is increasing or decreasing

(c) all local maxima and minima

Example: Using Calculus, find constants a and b so that the parabola $f(x) = x^2 + ax + b$ has a minimum at the point (3,5).

Example: The value of an investment at time t is given by $V(t)$. The rate of change, $V'(t)$, of the value of the investment is shown in the following figure.



(a) What are the critical points of the function $V(t)$?

(b) Identify each critical point as either a local maximum, a local minimum, or neither.

Example: Use the second derivative test to determine the local extrema for $g(t) = t + \frac{9}{t}$.

Section 4.2 Inflection Points

Test for Concavity
1. If $f'' > 0$ on (a,b), then the graph of f is **concave up** on (a,b).
2. If $f'' < 0$ on (a,b), then the graph of f is **concave down** on (a,b).

Definition A point $(c, f(c))$ on a curve is called an **inflection point** if the curve changes concavity at $(c, f(c))$.

Example Let $f(x) = x^4 - 4x^3 + 10$

(a) Find all critical points and local extremum.

(b) Find all inflection points and discuss the concavity of f.

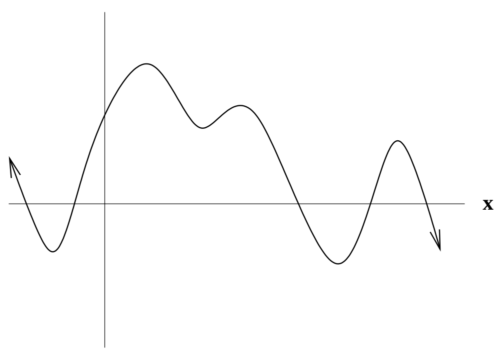
Example
Find the critical points and inflection points of the function $f(x) = xe^{-x}$.

Example (a) Water is flowing at a constant rate into a cylindrical container standing vertically. Sketch a graph showing the depth of water against time.

(b) What if the container is cone-shaped and is standing on its point.

Example: Sketch a graph of a function f with the following properties:
f has a critical point at $x = 3$ and $f'' = 0$ when $x = 7$;
f' is positive to the left of 3 and negative to the right of 3;
f'' is negative to the left of 7 and positive to the right of 7.

Example:



a) Suppose the graph above is the **original** function, $f(x)$. How many inflection points are there for the graph shown?

b) Suppose the graph above is of the **first derivative**, f'. How many inflection points would $f(x)$ have?

c) Suppose the graph above is of the **second derivative**, f''. How many inflection points would $f(x)$ have?

Section 4.3 Global Maxima and Minima

Global Extrema: Global Maximum and Global Minimum

$f(c)$ is a **global maximum** if $f(c)$ is greater than or equal to all values of f.
$f(c)$ is a **global minimum** if $f(c)$ is less than or equal to all values of f.

To find Global Extrema on a Closed Interval [a,b]

1. Find the critical values of f that are in the closed interval.
2. Evaluate f at these critical values and at the endpoints a and b.
3. The largest y-value from step 2 is the Global Maximum and the smallest is the Global Minimum.

To find Global Extrema on an Open Interval (a,b)

1. Find the critical values of f on the open interval.
2. Sketch a graph.

Example: Find the global extrema of $f(x) = x^3 - 9x^2 - 48x + 52$ on [-5,14].

Example: Find the global extrema of $f(x) = x^3 + 3x^2 - 1$ on [-3,2].

Example: Find the global extrema of $f(x) = 4x + \dfrac{1}{x}$ on [1,3].

Example: Graph a function with the given properties:
Has local and global minimum at $x = 3$, local and global maximum at $x = 8$.