Section 4.7 Logistic Curves

Example The following table gives the population of the United States in millions for some selected early years.

Year	1790	1810	1830	1850	1870	1890
Population	3.9	7.2	12.9	23.2	38.6	63.0

(a) Based on the data given for the years 1790 and 1890, find the best fitting exponential function using exponential regression. Determine the correlation coefficient. Graph. Using this model, estimate the population in 1990.

(b) Now find the best-fitting logistic curve. Graph. Using this model, estimate the population in 1990.

The Logistic Function
A logistic function has the form

$$P = f(t) = \frac{L}{1 + Ce^{-kt}}$$

where L, C, and K are positive constants.

Properties of the logistic function:

⋆ The value L represents the carrying capacity or upper bound for P.
⋆ The point of diminishing returns is the inflection point where P is growing the fastest. It occurs where $P = \frac{L}{2}$.
⋆ The logistic function is approximately exponential for small values of t with growth rate k.

Example A biologist found that the number of Drosophila fruit flies, $N(t)$, assumes the following growth pattern if the food source is limited. (t is in days.)

$$N(t) = \frac{400}{1 + 39e^{-0.4t}}$$

(a) How many fruit flies were there in the beginning?

(b) How many flies were there after 3 days?

(c) At what time was the population increasing most rapidly?

Example The following table shows the total sales (in thousands) of a new CD since it was introduced.

t(months)	0	1	2	3	4	5	6	7
P(total sales in thous.)	0.5	2	8	33	95	258	403	496

(a) Find the point where concavity changes in this function. Use it to estimate the maximum potential sales, L.

(b) Using a graphing calculator to find a logistic regression function for the given data. What maximum potential sales does this function predict?