Chapter F Finance

Section F.1 Simple Interest and Discount

Simple interest – interest computed as a percentage of the principal

Simple interest earned: $I = Prt$

> I=interest earned
> P=principal
> r=interest rate (decimal form)
> t=time period (in years)

Future value: $F = P + Prt = P(1 + rt)$

Example
If a bank loans $525 to an individual for 3½ years at 7.25% simple interest, what will be the amount repaid on the loan?

Example
If a $1000 deposit grows in value to $1024 after 9 months, what is the simple interest rate that is earned?

Example
How much should be place in an account that pays simple interest of 4% so that the value of the account after 18 months is $3000?

Example
You have an account with $500 that pays 5% simple interest. How long until your account doubles in value?

When a loan is ***discounted***, the interest owed is deducted when the loan is made. The interest deducted is called the ***discount*** (D), the amount the borrower actually receives is called the ***proceeds*** (P) and the amount to be repaid is called the ***maturity value*** (M). So

$$D = Mrt \text{ and } P = M - D$$

Where t is in years and r is the annual simple interest rate.

Example
A borrower gets a loan in which she agrees to pay the bank $4000 in 8 months at 6% simple discount. What is the discount and what are the proceeds?

Example

If a borrower wants to have proceeds of $4000 on a discounted loan for 8 months at 6%, what is the maturity value of the loan?

Example

A borrower gets a loan in which she agrees to pay the bank $4000 in 8 months at 6% simple discount. What is the effective interest rate?

The effective rate of interest on a discounted loan of length t years with a discount rate of r is $r_{eff} = \dfrac{r}{1 - rt}$

Section F.2 Compound Interest

When interest is paid periodically and the interest earns interest, we have compound interest.

Example
You deposit $100 into an account that pays 10% annual interest that is compounded annually. How much is in the account after 4 years?

$$F = P\left(1 + \frac{r}{m}\right)^{mt}$$

where m is the number of compounding periods per year and r is the annual interest rate as a decimal.

Example
You deposit $300 into an account that pays 6% annual interest compounded monthly. How much is in the account after 3 months?

If interest is compounded	Then m is
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

```
N=■
I%=0
PV=0
PMT=0
FV=1
P/Y=1
C/Y=1
PMT:END BEGIN
```

ON THE CALCULATOR, TVM Solver:

N is   # of comp. periods

I% is   interest rate as a percent

PV is   present value   ⎤
                         ⎬ $
PMT is   Payments   ⎬ must have
                         ⎬ a sign
FV is   future value   ⎦ change

P/Y and C/Y is   # of cmp periods per year (m)

set PMT: END (make payments at the end of the cycle)

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Example

You deposit $1000 into an account that pays 10% annual interest. How much is in the account after 10 years if

(a) the account earns simple interest?

$$\text{\# of comp periods} = N = m*t$$

$$F = 1000(1 + .1 \times 10) = \$2000$$

(b) the account earns interest compounded annually?

$N = 10 \times 1$ $PMT = 0$ solve $\$2593.74$
$I = 10$ $FV = ?$
$PV = 1000$ $P/Y = 1$

(c) the account earns interest compounded semiannually?

$N = 10 \times 2 = 20$ PMT Solve $\$2653.30$ ($\$59.56$ over once per year)
$I =$ $FV =$
$PV =$ $P/Y = 2$

(d) the account earns interest compounded quarterly?

$N = 4 \times 10 = 40$ $\$2685.06$ (incr of $\$31.76$ over 2× per year)
$P/Y = 4$

(e) the account earns interest compounded monthly?

$N = 120$ $\$2707.04$ (incr of $\$21.98$ over 4× per year)
$P/Y = 12$

(f) the account earns interest compounded weekly?

$N = 52 \times 10 = 520$ $\$2715.67$ (incr of $\$8.63$ over 12× per year)
$P/Y = 52$

(g) the account earns interest compounded daily?

$N = 365 \times 10 = 3650$ $\$2717.91$ (incr of $\$2.24$ over weekly)
$P/Y = 365$

Effective interest rate r_{eff} can be found with a FINANCE command

Eff(I%, m)

Example
You find a bank that pays 8.15% annual interest compounded quarterly and another that pays 8.1% annual interest compounded daily. Which is a better deal?

$$eff\,(8.15, 4) = 8.4025\%$$
$$eff\,(8.1, 365) = 8.4361\% \quad \text{\textcolor{red}{* better deal}}$$

If interest is compounded continuously, then the future value of an investment of P dollars at an annual interest of r (expressed as a decimal) for t years is $F = Pe^{rt}$.

Example
You deposit $1000 into an account that pays 10% annual interest. How much is in the account after 10 years if the interest is compounded continuously?

$$F = 1000\,e^{(.1 * 10)} = \$2718.28 \left(\begin{array}{c}\text{more of } \$0.37\\ \text{over daily}\end{array}\right)$$

Example

You wish to have \$60,000 available for your child's college expenses. You find an account that pays 7% annual interest compounded monthly.

How much needs to be deposited now to reach this goal if you want to have the money available in 18 years?

$N = 18 \times 12$ $PMT = 0$

$I = 7$ $FV = 60000$ $\nearrow$ \$17,081.66

$PV = ?$ $P/Y = 12$

How much interest was earned?

$60000 - 17081.66 = \$42,918.34$

What if you wanted the money available in 8 years?

$N = 8 \times 12$ $PMT = 0$

$I = 7$ $FV = 60000$ $\nearrow$ \$34,328.35

$PV = ?$ $P/Y = 12$

How much interest was earned?

$60000 - 34328.35 = \$25,671.65$

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Example

You deposit $4000 in an account that earns 5% annual interest compounded weekly. How long does it take for the account to double in value?

N = ?   → N = 721
I = 5
PV = ‾4000   ⟨ 721 weeks ⟩
PMT = 0
FV = 8000   fyi   just under 14 year
P/Y = 52

must be neg

Example

You deposited $200 in a savings account 10 years ago. It is now worth $350. You know the account paid annual interest that was compounded annually, but you don't remember the interest rate promised. What was the annual interest rate for the account?

N = 10   PMT = 0
I = ?   FV = 350   → 5.76%
PV = -200   P/Y = 1

Example

Your retirement account was worth $16,000 on January 1 and worth $15,000 two years later. Find the annual interest rate.

N = 2   PMT = 0
I = ?   FV = 15000   → -3.18%
PV = -16000   P/Y = 1

Section F.3 Annuities and Sinking Funds

Example

To save for retirement, you deposit $2000 every year in an account that earns 6% annual interest compounded annually.

(a) How much will you have after 10 years? How much interest did you earn?

$N = 10 \times 1 = 10$ $PMT = 2000$

$I = 6$ $FV = ?$

$PV = 0$ $P/y = 1$

$\boxed{\$ 26361.59}$ money deposited

$I = 26361.59 - 10 \times 2000$

$= \$6361.59$

(b) How much will you have after 25 years? How much interest did you earn?

$N = 25$

$FV = ?$

$\$ 109,729.02$ total in account

$- 25 \times 2000$

―――――――――

$\$ 59,729.02$ in interest

(c) How much will you have after 40 years? How much interest did you earn?

$N = 40$

$FV = ?$

$\$ 309,523.93$ in account

$- 40 \times 2000$

―――――――――

$\$ 229,523.93$

An **annuity** is an account to which regular payments are made. An annuity that is **certain and simple** has the following properties:
1. The payments are made at fixed time intervals
2. The periodic payments are of equal size
3. The payments are made at the end of the interval
4. The interest is paid at the end of the interval

Many loans and savings plans are certain and simple annuities

Any account that is established to meet a future need is called a
sinking fund.

Example
You wish to have $60,000 available for your child's college
expenses. You find an account that pays 7% annual interest
compounded monthly. How much needs to be deposited every
month to reach this goal if you want to have the money available in
18 years? Assume that the monthly payments are of equal size.
How much interest does the account earn?

$N = 12 \times 18 = 216$ $PMT = ?$ $\$139.30$ PmT each month

$I = 7$ $FV = 60000$ $60000 - 216(139.30)$

$PV = 0$ $P/Y = 12$ $= \$29,911.20$ un interest

Example
You start a savings account with the goal of saving $5000 in 4
years by making regular quarterly payments to an account that
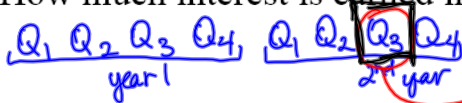earns 8% annual interest compounded quarterly.

(a) How large are the quarterly payments?

$N = 4 \times 4 = 16$ $PV = 0$ $FV = 5000$ ⎱ $\$268.25$
$I = 8$ $PmT = ?$ $P/Y = 4$ ⎰

(b) How much interest is earned in all?

$5000 - 16 \times 268.25 = \$708$

(c) How much interest is earned in the 3rd quarter of the 2nd year?

Q₁ Q₂ Q₃ Q₄, Q₁ Q₂ Q₃ Q₄
year 1   2ⁿᵈ year

End of 1st quarter : $268.25 \times \frac{.08}{4} = \$5.37$ in interest

End of 2nd quarter : $(268.25 + 268.25 + 5.37) \times \frac{.08}{4} = \$10.84$

How much is in the account after 7 payments?

Use: $N = 7, PV = 0, PmT = 268.25$ solve for FV

$\$1994.25$ in the account

$\times \frac{.08}{4} = \$39.89$ in interest

Example

A cello costs $574 to buy or $40.58 per month on a 2 year rent-to-own plan. What is the interest rate charged?

$N = 24$ $PMT = 40.58$

$I = ?$ $FV = 0$ $I = 56.96\%$

$PV = -574$ $P/Y = 12$

Example

You owe $6000 on a credit card that charges 21% annual interest compounded monthly on the unpaid balance.

(a) If you make monthly payments of $120, how long will it take to pay it off? *what you owe*

$N = ?$ $PV = -6000$ $FV = 0$ $N = 119.86$

$I = 21$ $PMT = 120$ $P/Y = 12$ 120 months

(b) How much interest is paid in all?

$120 \frac{\$}{mo} * 120\, mo = 14,400$

-6000

$\$8400$ paid on interest

TRY THIS (c) How much do you own after 5 years of making payments? *what we owe*

$N = 60$ $PV = ?$ $FV = 0$ $\$4435.68$

$I = 21$ $PMT = 120$ $P/Y = 12$

Example

You want a pair of diamond earrings that cost $800. You decide to stop buying daily lattes and instead save the $5 every day in an account that pays 6% annual interest compounded daily. How long until you have the money for the earring?

$\frac{800\,\$}{5\,\$/day} = 160$ days

$N = ?$ $N = 162$

$I = 6$

$PV = 0$

$PMT = 5$ $N = 158$ days

$FV = 800$

$P/Y = 365$

Quiz Tues on F.2 + F.3

Section F.4 Present Value of Annuities and Amortization

Example

When you retire you would like to have $30,000 per year. Assuming that you will need to have 20 payments <u>starting right now</u>, how much do you need to deposit into an account that pays 5% annual interest that is compounded annually?

$N = 19$ $PMT = 30000$ $PV = 362,559.63$ deposit $*$

$I = 5$ $FV = 0$ $+ \quad 30,000 \quad 1^{st} \; PMT$

$PV = ?$ $P/y = 1$

$\underline{}$

$\$392,559.63$

Example

You wish to purchase a house that costs $130,000. You make a down payment of $20,000 and <u>finance the remainder</u> for 30 years at 5.1% annual interest compounded monthly on the unpaid balance.

(a) How large are the monthly payments?

$N = 30 \times 12$ $PMT = ?$ ⟩⟩ $\$597.24$

$I = 5.1$ $FV = 0$

$PV = 110000$ $P/Y = 12$

(b) How much interest is paid in all?

$360 * \$597.24 = 215,006.40$

$\underline{ -110\,000}$

$\$105,006.40$

(c) How much of the first payment is interest?

$$I = 110,000 * \cdot \frac{.051}{12} = 467.50$$

$597.24 - 467.50 = 129.74$ to pay off the loan

(d) How much do you still owe the bank after your first payment?

$$110,000 - 129.74 = \$109,870.26$$

value of item

← EQUITY ——→ ← owe on loan →

(e) What is your equity after your first payment?

$$130,000 - 109,870.26 = 20,129.74$$

(f) What is your equity after your second payment?

Interest is $109,870.26 * \cdot {}^{.051}/_{12} = 466.95$

$597.24 - 466.95 = 130.29$ to pay off loan

Now owe $109,870.26 - 130.29 = 109,739.97$

$EQ = 130,000 - 109,739.97 = 20,260.03$

(g) What is your equity after your 12th payment?

✳ $N = 348$ pmts left $PMT = 597.24$ ⟶ $\$108,406.15$

$I = 5.1$ $FV = 0$ $EQ = 130,000 -$

$PV = ?$ $P/Y = 12$ $= 21,593.85$

Example

You borrow $3000 at 12% annual interest compounded quarterly for 2 years. Show in a table how much of each payment is interest, how much is paid towards the principal, how much of the loan is paid off and how much is still owed each quarter.

end of period	# PMT left	PMT ($)	interest ($)	towards principal	outstanding principal	equity
0						
1						
2						
3						
4						
5						
6						
7						
8						

Example

For the house purchased on page 13, fill in the following lines of an amortization table.

end of period	# PMT left	PMT ($)	interest ($)	towards principal	outstanding principal	equity
0						
1						
2						
12						
60						
120						
180						
240						
300						