MATH 141 SPRING 2008 QUIZ 15 NAME KEY

1. (3 points) A student has a quiz in his math class. He can study hard for the quiz, study a bit for the quiz or not study at all. The quiz can be challenging, moderate or easy. The student assigns a "satisfaction value" to each of the outcomes as shown in the matrix below:

$$\begin{array}{l c} & \begin{array}{ccc} \text{challenging} & \text{moderate} & \text{easy} \end{array} \\ \begin{array}{l} \text{study hard} \\ \text{study a little} \\ \text{don't study} \end{array} & \begin{pmatrix} 5 & 3 & 2 \\ 2 & 4 & 1 \\ -1 & 1 & 0 \end{pmatrix} \end{array}$$

a) What should the student do if he is an optimist?

studyhard (and expect a challengeng quiz)

b) What should the student do if he is a pessimist?

study hard (and expect a challenging quiz)

c) What is the expected value of studying a little if there is a 25% chance of a challenging quiz, 50% chance of a moderate quiz and 25% chance of an easy quiz?

$E = 2(.25) + 4(.5) + 1(.25) = 2.75$

2. (1 point) Remove any dominated strategies from the matrix $\begin{pmatrix} \cancel{2} & \cancel{3} & \cancel{-1} & \cancel{-5} \\ \cancel{6} & 5 & 4 & 1 \\ \cancel{1} & -4 & 0 & 3 \end{pmatrix}$

$$\begin{bmatrix} 5 & 4 & 1 \\ -4 & 0 & 3 \end{bmatrix}$$

3. (1 point) Bill and Sue play a game with coins. Both flip a coin at the same time. If both coins show heads or both coins show tails, Bill wins $2 from Sue. If one coin shows heads and one shows tails, Sue wins $2 from Bill. Construct the payoff matrix for this game.

$$\begin{array}{cc} & \text{Sue} \\ & \begin{array}{cc} H & T \end{array} \\ \text{Bill} \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \end{array}$$

or

$$\begin{array}{cc} & \text{Bill} \\ & \begin{array}{cc} H & T \end{array} \\ \text{Sue} \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \end{array}$$

4. (2 points) Using the payoff matrix below for a two-person zero-sum game, find the saddle point, the optimal strategy for the row player and the value of the game. Is the game fair? If not, why not?

$$\begin{pmatrix} 1 & -4 & 5 \\ -2 & 0 & -1 \\ 4 & 1 & 1 \\ -1 & -2 & -4 \end{pmatrix}$$

Saddle @ row 3 and col 2
row player plays row 3 all the time
Value of the game is 1
Not fair as it favors 1

5. (3 points) Given the payoff matrix $M = \begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix}$,

(a) What is the value of this game if the row player plays row 1 20% of the time and the column player plays column 1 70% of the time?

(b) What is the row player's optimum strategy?

(c) What is the value of the game if each player plays their optimum strategy?

(a) $(.2 \quad .8)\begin{bmatrix} 2 & -3 \\ -5 & 6 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} = [-1.26]$

Value is -1.26

(b) $p_1 = \dfrac{6-(-5)}{2+6-(-3)-(-5)} = \dfrac{11}{16} \doteq .6875$

$p_2 = 1 - p_1 = \dfrac{5}{16} = .3125$

Play row 1 .6875 of the time and row 2 .3125 of the time.

(c) $q_1 = \dfrac{6-(-3)}{2+6-(-3)-(-5)} = \dfrac{9}{16} = .5625$

$q_2 = 1 - q_1 = 7/16 = .4375$

$E = (.6875 \quad .3125)\begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} .5625 \\ .4375 \end{pmatrix} \approx (-.1875)$

$p_1 = \dfrac{a_{22}-a_{21}}{a_{11}+a_{22}-a_{12}-a_{21}}$ ~~and $p_2 = 1 - p_1$~~ $\qquad q_1 = \dfrac{a_{22}-a_{12}}{a_{11}+a_{22}-a_{12}-a_{21}}$ ~~and $q_2 = 1 - q_1$~~