CHAPTER 16: IDENTIFICATION NUMBERS

Recognize any of these? (A) Yes (B) No

Clicker Question

979-845-3261 77843-3368

876-87-6543 978-0-495-83538-7

16.1 Check Digits

Identification numbers may or may not have information coded in.

Identification numbers are subject to errors.

Consider your exam score as a two digit number a_1a_2.

What if a score of 75 was entered as 65? Or as a 57? Add a ***check digit*** to catch some types of errors.

Exam score is entered as $a_1a_2a_3$ where $a_3 = a_1 + a_2 \bmod 10$

EXAMPLE

Find the check digit $a_3 = a_1 + a_2 \pmod{10}$ for the following exam scores:

75 $7+5=12$ & $\frac{12}{10} = 1\ R2$, $a_3 = 2 \Rightarrow$ 752

15 652 valid

65 $6+5=11$ 651

57 Check digit is

(A) 0 (B) 1 (C) 2 (D) 12 (E) None of these

572

More about modular arithmetic

EXAMPLE

$= 3\times 7+2$

(a) If today is Tuesday, what day of the week is it in 23 days?

Thursday

(b) If it is 7A, what time is it in 15 hours? 10P

Definition: Congruence Modulo m

Let a, b, and m be integers with $m \geq 2$. Then a is congruent to b modulo m, written

$$a \equiv b \bmod m$$

means that m evenly divides $a-b$.

EXAMPLE

Determine if each of the congruences below are true or false.

(a) $24 \equiv 0 \bmod 3$ $\frac{24-0}{3} = 8R0$ TRUE

(b) $21 \equiv 1 \bmod 5$ $\frac{21-1}{5} = 4R0$ TRUE

(c) $27 \equiv 5 \bmod 11$ $\frac{27-5}{11} = 2R0$ TRUE

(d) $21 \equiv 1 \bmod 7$ **(A) True** **(B) False** **(C) I'm lost**

$\frac{21-1}{7} = \frac{20}{7} = 2R6$

32 mod 38 $= 0R32$

EXAMPLE

Find the following values

(a) 99 mod 11 is 0

$\frac{99}{11} = 9R0$

(b) 12 mod 7 is 5

$\frac{12}{7} = 1R5$

(c) 27 mod 8 is 3

(d) 40 mod 13 is 1

Types of errors when dealing with identification numbers:

- Replacing one digit with a different digit (single digit error)
  - *ac* entered rather than *ab*
- Transposing two adjacent digits (adjacent transposition error)
  - *ba* entered rather than *ab*
- Transposing a sequence of digits (jump transposition error)
  - *cba* entered rather than *abc*

We found that using the check digit $a_3 = a_1 + a_2$ (mod 10) for exam scores did not detect transposition errors. Would using a different mod number help? No. TRY MOD 5

$75 \Rightarrow 7+5 = 12$, $\frac{12}{5}$ has $R = 2$ so 752

$25 \Rightarrow 2+5 = 7$, $\frac{7}{5}$ has $R = 2$ so 252

You can assign a ***weight*** to digits in a code. That is, multiply one or more of the digits in a code by an integer.

EXAMPLE

Using the check digit $a_3 = 2a_1 + a_2 \bmod 10$, determine if the following exam scores are valid

(a) 759 $a_3 = 2(7) + 5 = 19 \bmod 10 = 9$ valid score

(b) 657 $2(6)+5 = 17 \bmod 10 = 7$ Valid

(c) 577 $2(5)+7 = 17 \bmod 10 = 7$ valid

(d) 679 $2(6)+7 = 19 \bmod 10 = 9$ valid

$a_1 = 6$

$a_1 = 1$

$2(6-1) = 10$

SDE didn't catch

(e) 179 This score is

(A) valid (B) not valid (C) not sure

$2(1)+7 = 9$, $\frac{9}{10} = 0 R 9$

What is the problem here?

$2 * 5 = 10$ so if the wrong digits differ by 5 it won't get caught

EXAMPLE

Will the check digit $a_3 = 3a_1 + a_2 \bmod 10$ catch single digit errors and adjacent transposition errors? What if we did mod 11?

catches all

SDE in 1st position? That is will $a_1 a_2$ have the same ck digit as $b_1 a_2$?

$a_3 = 3a_1 + a_2$ and $b_3 = 3b_1 + a_2 \Rightarrow$ is $a_3 - b_3 \bmod 10 = 0$?

$a_3 - b_3 = 3a_1 + a_2 - (3b_1 + a_2)$
$= 3a_1 + a_2 - 3b_1 - a_2 = 3a_1 - 3b_1 = \boxed{3(a_1 - b_1) = 0 \bmod 10}$

0, 10, 20, 30, 40

$|a_1 - b_1| = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

no error

will catch all SDE in 1st position

SDE in 2nd position: $a_1 a_2$ entered as $a_1 b_2$

$a_3 - b_3 = 0 \bmod 10$?

$(3a_1 + a_2) - (3a_1 + b_2) = 3a_1 + a_2 - 3a_1 - b_2 = a_2 - b_2 = 0 \bmod 10$?

will catch all SDE in 2nd position

ADJ TR ERROR: $a_1 a_2$ entered as $a_2 a_1$

$a_3 = 3a_1 + a_2$ and $b_3 = 3a_2 + a_1$

$a_3 - b_3 = 3a_1 + a_2 - 3a_2 - a_1 = 2a_1 - 2a_2 = 2(a_1 - a_2)$

catches all but the case $|a_1 - a_2| = 5$

Some check digit methods

- US Postal Service Money order: $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}|a_{11}$. The check digit is ch digit
  $a_{11} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} \bmod 9$
- American Express and Visa traveler's checks along with Euro banknotes have a check digit that is chosen to make the sum of all the digits evenly divisible by 9.
- UPC: $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}$ has a_{12} chosen so that the sum $3(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11}) + 1(a_2 + a_4 + a_6 + a_8 + a_{10})$ is divisible by 10.
- ISBN – see your textbook
- Bank routing numbers have 8 digits and a check digit at the end, $a_1a_2a_3a_4a_5a_6a_7a_8a_9$. The check digit a_9 is the last digit of the sum $9(a_3 + a_6) + 7(a_1 + a_4 + a_7) + 3(a_2 + a_5 + a_8)$

EXAMPLE

(a) Determine the check digit for a US Postal Service Money order with identification number 7234541780X

1 2 3 4 5 6 7 8 9 10

$7+2+3+4+5+4+1+7+8+0=41$

$41 \bmod 9 = 5$ is check digit

(b) Determine the check digit for the bank routing number 09100001.

1 2 3 4 5 6 7 8

$9(1+0)+7(0+0+0)+3(9+0+1)=39$ ch digit

(c) What is the check digit x in the American Express traveler's check with number 042503679x ?

36+ x is divisible evenly by 9

(A) 0 (B) 3 (C) 6 (D) 8 (E) none of these

makes the sum of all numbers divisible by 9

EXAMPLE 23 mod 7 = 2 || Sum 16, 23, 30, 37

Suppose a check digit is assigned to a 4 digit number by appending the sum of the 4 digits mod 7 to the end. If the number 9680|2 has a single digit error, but the check digit is correct, what might the correct number?

x680 = 14 + x ⇒ x = 2 will make the sum 16, $\frac{16}{7}$ = 2R2 ⇒ ck dg is 2

9x80 = 17 + x since only 6 works there's no error

96x0 = 15 + x ⇒ x = 1 will make the sum 16 9610

968x = 23 + x ⇒ x = 7 30 9687

Credit Cards

A 15 digit number with a 16th number as the check digit,

$a_1a_2a_3a_4\ a_5a_6a_7a_8\ a_9a_{10}a_{11}a_{12}\ a_{13}a_{14}a_{15}a_{16}$

The check digit is found by adding all the numbers in the odd positions and doubling that,

$$s_1 = 2(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13} + a_{15})$$

Then count the number of digits in the odd positions that are over 4 and call this s_2.

Next, find the sum of the numbers in the even positions,

$$s_3 = a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12} + a_{14}$$

The check digit a_{16} is the number needed to bring the total of the three sums above to a multiple of 10.

EXAMPLE

(a) Is 4128 0012 3456 789$\boxed{0}$ a valid credit card number?

(A) Yes, it is valid (B) Not valid (C) Not sure

$S_1 = 2(4+2+0+1+3+5+7+9) = 2 \times 31 = 62$

$S_2 = 3$

$S_3 = 1+8+0+2+4+6+8 = 29$

$62+3+29 = 94$

$S_1 + S_2 + S_3 + \text{ck digit} = \text{multiple of } 10$

(b) Suppose that a credit card number is $\overbrace{4264\ 5200\ 2177\ x}^{0,1,2,3,4,5,6,7,8,9}3\boxed{3}$7. What is the value of x?

this is the check digit

$S_1 = 2(4+6+5+0+2+7+x+3) = 54+2x$

$S_2 = 3$ or 4

$S_3 = 2+4+2+0+1+7+3 = 19$

$*$ 73, 83, 93, 103

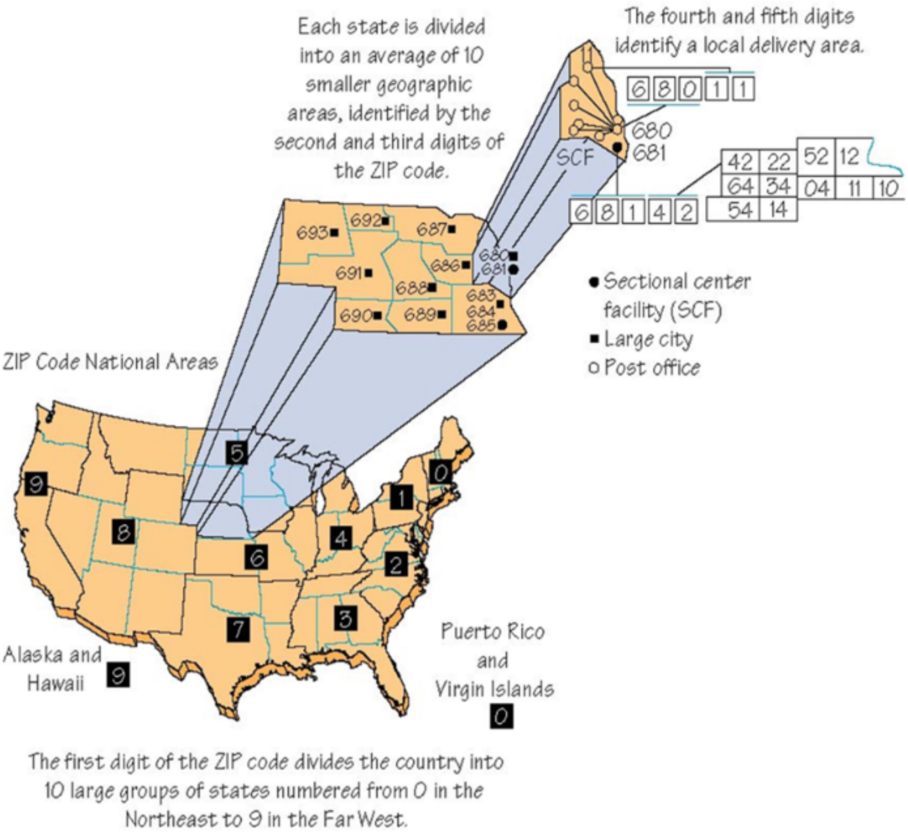
$S_1+S_2+S_3 = 54+2x+3+19$ if $x = \overbrace{0,1,2,3,4}$ — $76+2x$

$S_1+S_2+S_3 = 54+2x+4+19$ if $x = 5,6,7,8,9$ — $77+2x$

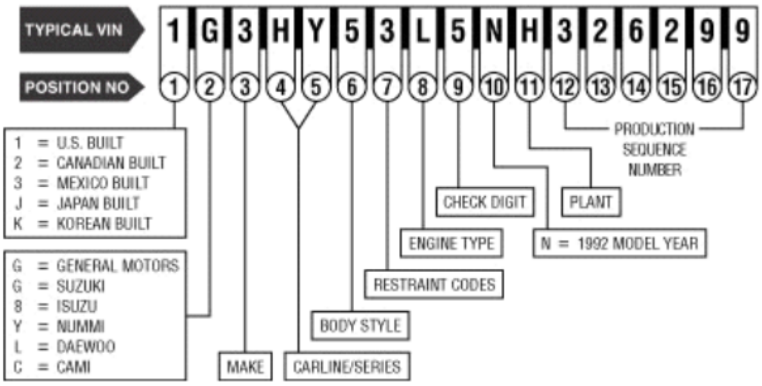
$*$

sum to be a multiple of 10

16.2 The Zip Code (and Other Codes with information)



VIN codes are Vehicle Identification Codes.



UPC is a universal product code.

First digit	Type of item
0	General groceries
2	Items sold by weight
5	Coupons

Next 5 digits are for the manufacturer, then 5 digits for the product number. The last digit is the check digit.

<u>16.3 Bar Codes</u>

A ***bar code*** is a series of dark bars and light spaces that represent characters.

Any system for representing data with only two symbols is ***a binary code***.

The ***postnet*** code is used to encode ZIP + 4 numbers by assigning the 10 digits to bar codes that have 5 vertical bars (2 long and 3 short).

There are 52 vertical bars needed. The first and last bars are *guard bars* to mark the beginning and end of the code. The 50 remaining bars give 10 digits. The first 9 of which are the ZIP + 4 and the last one is a check digit.

The postnet check digit is determined by adding the first 9 digits and making the 10th digit have the sum come to a multiple of 10.

The ***delivery-point*** barcode allows for two more digits so that the mail can be sorted in the order that it will be delivered from the carrier.

decode this

77843 | 3366 | 99 | 5 ck digit

The ***intelligent mail*** barcode uses 65 vertical bars to convert 31 digits of data. The bars have 3 lengths and can be in different vertical positions. The data encoded has the type of service, the mail owner, a unique tracking number and delivery zip.

ribbs.usps.gov/onecodesolution/

QR (Quick Response) codes

These can encode much more information and are popular in print media. The Cooking Light magazine from September, 2011 had quite a few, including

cuisinart.com/scan

16.4 Encoding Personal Data

Feb1 : $40(2-1)+1+500=541$
Jan1 : $40(1-1)+1+500=501$

In Florida, the last three digits of the driver's license number of a female with birth month m and birth date b are $40(m-1)+b+500$.

In Florida, the last three digits of the driver's license number of a male with birth month m and birth date b are $40(m-1)+b$. Jan 1 : 001

For both males and female in Florida the 4th and 5th digits from the end of the driver's license number give the year of birth.

EXAMPLE
Determine the last 5 digits of a Florida driver's license number for the following people

(a) A female born on July 18, 1942 4 2 7 5 8 end

$40(7-1)+18+500$

(b) A male born on May 1, 1988

$40(m-1)+b = 40(5-1)+1$
$= 161$

(A) 88201 (B) 88161 (C) 88121 (D) None of these

(c) What do you know about a person who has 61528 as the last 4 digits of their FL driver's license?

028
Jan 28th

Female
born 1961

(d) What about 34475?

Dec 31
$40(12-1)+31=471$

male
1934

fake D