Test I

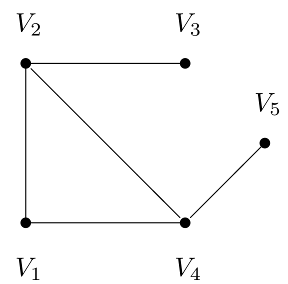
Instructions: *Show all work in your bluebook. Calculators that do linear algebra or calculus are* not *allowed.*

1. Define the following:

   (a) **(5 pts.)** $C[a,b]$, and its operations of addition and scalar multiplication.

   (b) **(5 pts.)** $\mathrm{Span}\{\mathbf{v}_1,\ldots,\mathbf{v}_\mathrm{n}\}$.

2. **(10 pts.)** Find the adjacency matrix A for the graph below, and compute the first row of A^2. What do these entries tell you about walks of length 2 that start from V_1?



3. **(20 pts.)** A linear system $A\mathbf{x}=\mathbf{b}$ has the augmented matrix $[A|\mathbf{b}]$ given below. Use row reduction to solve the system. Also, identify the leading variables and free variables, and find $N(A)$.

$$[A|b] = \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & -2 \\ 3 & -6 & 2 & 1 & 1 \\ -2 & 4 & -2 & -2 & 4 \end{array}\right)$$

4. **(10 pts.)** Let $S=\{(x_1\ x_2\ x_3)^T\in\mathbb{R}^3\,|\,x_1-2x_2=x_3\}\subset\mathbb{R}^3$. Determine whether or not S is a subspace of $\mathbb{R}^3$.

5. Let $C = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 1 \\ -2 & -5 & 3 \end{pmatrix}$.

   (a) **(15 pts.)** Find C^{-1} by row reducing the augmented matrix $[C|I]$, keeping careful track of the row operations that you use.

   (b) **(10 pts.)** By inspecting these row operations, give elementary matrices E, E', E'' such that $E''E'EC=U$, where U is upper triangular.

   (c) **(10 pts.)** Find $\det C$, using any method.

6. **(15 pts.)** Do <u>one</u> of the following problems.

(a) Define the term *inverse* of an $n \times n$ matrix A. Show that if A and B are invertible, then AB is, too, and $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Let A be an $n \times n$ matrix. Show that if $A\mathbf{x} = \mathbf{0}$ has *only* $\mathbf{x} = \mathbf{0}$ as a solution, then A is row equivalent to the identity.

(c) Let A be an $n \times n$ matrix. Show that if A is row equivalent to the identity, then A is nonsingular.