Test I

Instructions: *Show all work in your bluebook. Cell phones, laptops, calculators that do linear algebra or calculus, and other such devices are* not *allowed.*

1. **(10 pts.)** Find both the parametric equation for the plane passing through the three points $P(0,1,-1)$, $Q(1,1,2)$, $R(1,2,0)$ and the area of the triangle $\triangle PQR$.

2. **(10 pts.)** Let $\mathbf{v} = (1,-2,3,1)$ and $\mathbf{u} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Find the projection of $\mathbf{v}$ onto $\mathbf{u}$ and find the distance from $\mathbf{v}$ to the line $\mathbf{x} = t\mathbf{u}$.

3. Let $A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 3 & 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$.

   (a) **(10 pts.)** For the system $A\mathbf{x} = \mathbf{b}$, form the augmented matrix $[A|\mathbf{b}]$ and determine its reduced row echelon form.

   (b) **(5 pts.)** What are $\text{rank}(A)$, $\text{rank}([A|\mathbf{b}])$? Which are the leading columns of A?

   (c) **(5 pts.)** Is the system consistent or inconsistent? If the system is consistent, find the parametric form of the solution.

4. **(10 pts.)** Hourly temperature readings from five remote stations are recorded as 5×1 column vectors. Data analysis shows almost all of these vectors are linear combinations of the vectors in

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 4 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 2 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

   Determine whether $\mathbf{T} = (1\ 3\ -2\ 0\ 2)^T$ can be represented in this way. If so, find three numbers that represent this vector. Are these numbers unique?

5. **(10 pts.)** Use row reduction either to find C^{-1} or to show that it does not exist, given that the matrix C is

$$C = \begin{pmatrix} 1 & 3 & -1 \\ -1 & -4 & 3 \\ 2 & 7 & -4 \end{pmatrix}.$$

6. Let $B = \begin{pmatrix} -1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \\ 0 & 2 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}.$

   (a) **(10 pts.)** Use any method to evaluate $\det(B)$.

   (b) **(5 pts.)** Let $\mathbf{b} = (-1\ 0\ 0\ 1)^T$. Use Cramer's rule to find the value of x_4 in the solution to $B\mathbf{x} = \mathbf{b}$.

   (c) **(5 pts.)** What is the rank of B? Are the *columns* of B LI or LD? Explain.

7. **(10 pts.)** Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $L(\vec{x}) = (3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \times \vec{x}$. Show that L is linear and find the matrix 3×3 matrix A that represents L.

8. **(10 pts.)** Let $\mathcal{M}_{2\times2}$ be the set of 2×2 matrices, $A = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$. Determine whether or not $S = \{A \in \mathcal{M}_{2\times2} \,|\, y = 2z\}$ is a subspace of $\mathcal{M}_{2\times2}$.