Math 311-Zhou **Review for Exam III**

$u=(u_1,u_2,u_3), v=(v_1,v_2,v_3)$. To be able to compute $u\times v$.

$\|u\times v\|$ =the area of the parallelogram formed by the vectors u and v, $u\times v\perp u, v$.

Path: $\chi:[a,b]\to\mathbb{R}^3, \chi(t)=(x(t),y(t),z(t)), v(t)=\chi'(t)=(x'(t),y'(t),z'(t))$.

Arclength: $s(t)=\displaystyle\int_a^t \|v(\tau)\|d\tau, \frac{ds}{dt}=\|v(t)\|$ where $\|v(t)\|=[|x'(t)|^2+|y'(t)|^2+|z'(t)|^2]^{1/2}$.

Scalar filed: $f:X\subset\mathbb{R}^3\to\mathbb{R}$, Vector field: $F:X\subset\mathbb{R}^3\to\mathbb{R}^3, F=(F_1,F_2,F_3)$.

Gradient field: $\nabla f=(f'_x,f'_y,f'_z)$, Divergence: $\operatorname{div} F=\nabla\cdot F=\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}$, Curl $F=\nabla\times F$.

$\nabla\cdot F\equiv 0$ (incompressible), $\nabla\times F\equiv 0$ (irrotational). $\operatorname{curl}(\nabla f)\equiv 0$, $\operatorname{div}(\operatorname{curl} F)\equiv 0$.

Line integral along a path $\chi(t):I=[a,b]\to\mathbb{R}^3,\ \chi(t)=(x(t),y(t),z(t)),\ ds=\|\chi'(t)\|dt,\ d\vec{s}=\chi'(t)dt$.

Scalar line integral: $\displaystyle\int_\chi f ds=\int_a^b f(\chi(t))\|\chi'(t)\|dt$.

Vector line integral: $\displaystyle\int_\chi F\cdot d\vec{s}=\int_a^b F(\chi(t))\cdot\chi'(t)dt=\int_a^b Mdx+Ndy+Pdz\ \ (F=(M,N,P))$

Path independence: $\displaystyle\int_{c_1}F\cdot d\vec{s}=\int_{c_2}F\cdot d\vec{s}$ for any curves c_1,c_2 with the same initial and terminal points;

$\Longleftrightarrow \displaystyle\int_c F\cdot d\vec{s}=0$ for any closed curve c; $F=\nabla f$ (F =conservative (gradient field), f =scalar potential)

$\Longleftrightarrow \nabla\times F=0\Longrightarrow \displaystyle\int_{c_1}F\cdot d\vec{s}=f(B)-f(A)$. In $\mathbb{R}^2$, $\nabla\times F=0\Longleftrightarrow \frac{\partial N}{\partial x}=\frac{\partial M}{\partial y}$.

Green's Theorem $F(x,y)=(M(x,y),N(x,y))$, $\displaystyle\iint_D\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy=\int_C Mdx+Ndy$ where $C=\partial D$.

$\displaystyle\frac{1}{2}\int_C -ydx+xdy=$ the area of D.

Given $F=(F_1,F_2,F_3)$ conservative, find f s.t. $F=\nabla f$. Equating $f=\begin{cases}\int F_1dx+A(y,z)\\ \int F_2dy+B(x,z)\\ \int F_3dz+C(x,y)\end{cases}$ and get f.

Then evaluate $\displaystyle\int F\cdot d\vec{s}$ along an unknown curve.

Surface integral. Surface $S:\chi(s,t)=(x(s,t),y(s,t),z(s,t)),(s,t)\in D$. $N(s,t)=T_s\times T_t$ is the normal vector of S at $\chi(s,t)$ where $T_s=(x_s,y_s,z_s),\ T_t=(x_t,y_t,z_t)$.

Equation for the plane tangent to the surface S at $\chi(s,t)$: $N(s,t)\cdot((x,y,z)-\chi(s))=0$.

Denote $n(s,t)=N(s,t)/\|N(s,t)\|$, $dS=\|N(s,t)\|dsdt,\ d\vec{S}=N(s,t)dsdt$.

Surface area of $S=\displaystyle\iint_S dS=\iint_D\|T_s\times T_t\|dsdt$.

Scalar surface integral of f on S: $\displaystyle\iint_S fdS=\iint_D f(\chi(s,t))\|T_s\times T_t\|dsdt$.

Vector surface integral of F on S: $\displaystyle\iint_S F\cdot d\vec{S}=\iint_D F(\chi(s,t))\cdot N(s,t)dsdt$.

When $S:z=g(x,y),(x,y)=(s,t)\in D$. $N(s,t)=(-g_x(s,t),-g_y(s,t),1)$, $\|T_s\times T_t\|=\sqrt{g_x^2+g_y^2+1}$.

Divergence Theorem: $\displaystyle\iiint_V\nabla\cdot FdV=\iint_S F\cdot n\,dS\ \Big(=\iint_D F(\chi(s,t))\cdot N(s,t)dsdt\Big)$ where $S=\partial V$.

Stokes' Theorem: $\displaystyle\iint_S(\nabla\times F)\cdot d\vec{S}\Big(=\iint_D(\nabla\times F(\chi(s,t))\cdot N(s,t)dsdt\Big)=\int_C F\cdot d\vec{s}\Big(=\int_I F(\chi(t))\cdot\chi'(t)dt\Big)$; where $C=\partial S$.