8. Consider a linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 2 & 1 & | & 1 \\ -1 & 4 & 3 & | & 2 \\ 2 & -2 & a & | & 3 \end{bmatrix}$$

For what values of a will the system have a unique solution?

9. Consider a linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 2 & 5 & 3 & | & 0 \\ -1 & 1 & \beta & | & 0 \end{bmatrix}$$

(a) Is it possible for the system to be inconsistent? Explain.

(b) For what values of β will the system have infinitely many solutions?

10. Consider a linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 1 & 2 & 4 & | & 3 \\ 1 & 3 & a & | & b \end{bmatrix}$$

(a) For what values of a and b will the system have infinitely many solutions?

(b) For what values of a and b will the system be inconsistent?

11. Given the linear systems

(a) $x_1 + 2x_2 = 2$
    $3x_1 + 7x_2 = 8$

(b) $x_1 + 2x_2 = 1$
    $3x_1 + 7x_2 = 7$

solve both systems by incorporating the right-hand sides into a 2×2 matrix B and computing the reduced row echelon form of

$$(A|B) = \begin{bmatrix} 1 & 2 & | & 2 & 1 \\ 3 & 7 & | & 8 & 7 \end{bmatrix}$$

12. Given the linear systems
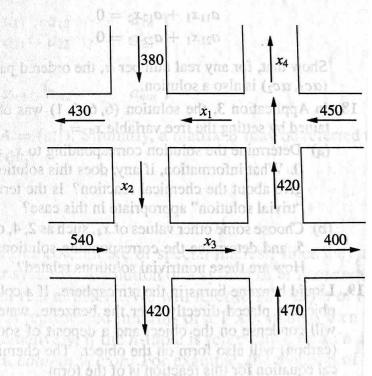
(a) $x_1 + 2x_2 + x_3 = 2$
    $-x_1 - x_2 + 2x_3 = 3$
    $2x_1 + 3x_2 = 0$

(b) $x_1 + 2x_2 + x_3 = -1$
    $-x_1 - x_2 + 2x_3 = 2$
    $2x_1 + 3x_2 = -2$

solve both systems by computing the row echelon form of an augmented matrix $(A|B)$ and performing back substitution twice.

13. Given a homogeneous system of linear equations, if the system is overdetermined, what are the possibilities as to the number of solutions? Explain.

14. Given a nonhomogeneous system of linear equations, if the system is underdetermined, what are the possibilities as to the number of solutions? Explain.

15. Determine the values of x_1, x_2, x_3, and x_4 for the following traffic flow diagram:

16. Consider the traffic flow diagram that follo where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are fixed posi integers. Set up a linear system in the unknowns x_2, x_3, x_4 and show that the system will be cor tent if and only if

$$a_1 + a_2 + a_3 + a_4 = b_1 + b_2 + b_3 + b_4$$

What can you conclude about the number of