The proofs are left for the reader (see Exercises 9 and 10 at the end of the section). ■

In the next section, we look at the effect of row operations on the value of the determinant. This will allow us to make use of Theorem 1.3 to derive a more efficient method for computing the value of a determinant.

SECTION 1 EXERCISES

1. Let

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ 2 & 3 & 2 \end{bmatrix}$$

(a) Find the values of $\det(M_{21})$, $\det(M_{22})$, and $\det(M_{23})$.

(b) Find the values of A_{21}, A_{22}, and A_{23}.

(c) Use your answers from part (b) to compute $\det(A)$.

2. Use determinants to determine whether the following 2×2 matrices are nonsingular:

(a) $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -6 \\ 2 & 4 \end{bmatrix}$

3. Evaluate the following determinants:

(a) $\begin{vmatrix} 3 & 5 \\ -2 & -3 \end{vmatrix}$ (b) $\begin{vmatrix} 5 & -2 \\ -8 & 4 \end{vmatrix}$

(c) $\begin{vmatrix} 3 & 1 & 2 \\ 2 & 4 & 5 \\ 2 & 4 & 5 \end{vmatrix}$ (d) $\begin{vmatrix} 4 & 3 & 0 \\ 3 & 1 & 2 \\ 5 & -1 & -4 \end{vmatrix}$