1. Consider the vectors $\mathbf{x}_1 = (8,\ 6)^T$ and $\mathbf{x}_2 = (4,\ -1)^T$ in R^2.

(a) Determine the length of each of the vectors.

(b) Let $\mathbf{x}_3 = \mathbf{x}_1 + \mathbf{x}_2$. Determine the length of $\mathbf{x}_3$. How does its length compare to the sum of the lengths of $\mathbf{x}_1$ and $\mathbf{x}_2$?

(c) Draw a graph illustrating how $\mathbf{x}_3$ can be constructed geometrically using $\mathbf{x}_1$ and $\mathbf{x}_2$. Use this graph to give a geometrical interpretation of your answer to the question in part (b).

2. Repeat Exercise 1 for the vectors $\mathbf{x}_1 = (2,\ 1)^T$ and $\mathbf{x}_2 = (6,\ 3)^T$.

3. Let C be the set of complex numbers. Define addition on C by

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and define scalar multiplication by

$$\alpha(a + bi) = \alpha a + \alpha bi$$

for all real numbers α. Show that C is a vector space with these operations.

4. Show that $R^{m\times n}$ with the usual addition and scalar multiplication of matrices satisfies the eight axioms of a vector space.

5. Show that $C[a, b]$ with the usual scalar multiplication and addition of functions satisfies the eight axioms of a vector space.

6. Let P be the set of all polynomials. Show that P with the usual addition and scalar multiplication of functions forms a vector space.

7. Show that the element $\mathbf{0}$ in a vector space is unique.

8. Let $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ be vectors in a vector space V. Prove that if

$$\mathbf{x} + \mathbf{y} = \mathbf{x} + \mathbf{z}$$

then $\mathbf{y} = \mathbf{z}$.

9. Let V be a vector space and let $\mathbf{x} \in V$. Show that:

(a) $\beta\mathbf{0} = \mathbf{0}$ for each scalar β.

(b) If $\alpha\mathbf{x} = \mathbf{0}$, then either $\alpha = 0$ or $\mathbf{x} = \mathbf{0}$.

10. Let S be the set of all ordered pairs of real numbers. Define scalar multiplication and addition on S by

$$\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2)$$

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)$$

We use the symbol $\oplus$ to denote the addition operation for this system in order to avoid confusion with the usual addition