SECTION 2 EXERCISES

1. Refer to Exercise 1 of Section 1. For each linear transformation L, find the standard matrix representation of L.

2. For each of the following linear transformations L mapping $\mathbb{R}^3$ into $\mathbb{R}^2$, find a matrix A such that $L(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^3$:
   (a) $L((x_1, x_2, x_3)^T) = (x_1 + x_2, 0)^T$
   (b) $L((x_1, x_2, x_3)^T) = (x_1, x_2)^T$
   (c) $L((x_1, x_2, x_3)^T) = (x_2 - x_1, x_3 - x_2)^T$

3. For each of the following linear operators L on $\mathbb{R}^3$, find a matrix A such that $L(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^3$:
   (a) $L((x_1, x_2, x_3)^T) = (x_3, x_2, x_1)^T$
   (b) $L((x_1, x_2, x_3)^T) = (x_1, x_1 + x_2, x_1 + x_2 + x_3)^T$
   (c) $L((x_1, x_2, x_3)^T) = (2x_3, x_2 + 3x_1, 2x_1 - x_3)^T$

4. Let L be the linear operator on $\mathbb{R}^3$ defined by

$$L(\mathbf{x}) = \begin{Bmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{Bmatrix}$$

   Determine the standard matrix representation A of L, and use A to find $L(\mathbf{x})$ for each of the following vectors $\mathbf{x}$:
   (a) $\mathbf{x} = (1, 1, 1)^T$ (b) $\mathbf{x} = (2, 1, 1)^T$
   (c) $\mathbf{x} = (-5, 3, 2)^T$

5. Find the standard matrix representation for each of the following linear operators:
   (a) L is the linear operator that rotates each $\mathbf{x}$ in $\mathbb{R}^2$ by 45° in the clockwise direction.