11. Find the distance from the point (2, 1, −2) to the plane

$$6(x-1)+2(y-3)+3(z+4)=0$$

12. Prove that if $\mathbf{x} = (x_1, x_2)^T$, $\mathbf{y} = (y_1, y_2)^T$, and $\mathbf{z} = (z_1, z_2)^T$ are arbitrary vectors in $\mathbb{R}^2$, then

(a) $\mathbf{x}^T\mathbf{x} \geq 0$ **(b)** $\mathbf{x}^T\mathbf{y} = \mathbf{y}^T\mathbf{x}$

(c) $\mathbf{x}^T(\mathbf{y}+\mathbf{z}) = \mathbf{x}^T\mathbf{y} + \mathbf{x}^T\mathbf{z}$

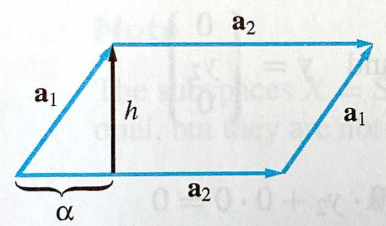
13. Show that if **u** and **v** are any vectors in $\mathbb{R}^2$, then $\|\mathbf{u}+\mathbf{v}\|^2 \leq (\|\mathbf{u}\| + \|\mathbf{v}\|)^2$ and hence $\|\mathbf{u}+\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$. When does equality hold? Give a geometric interpretation of the inequality.

14. Let $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ be vectors in $\mathbb{R}^3$. If $\mathbf{x}_1 \perp \mathbf{x}_2$ and $\mathbf{x}_2 \perp \mathbf{x}_3$, is it necessarily true that $\mathbf{x}_1 \perp \mathbf{x}_3$? Prove your answer.

15. Let A be a 2×2 matrix with linearly independent column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. If $\mathbf{a}_1$ and $\mathbf{a}_2$ are used to form a parallelogram P with altitude h (see the accompanying figure), show that

(a) $h^2\|\mathbf{a}_2\|^2 = \|\mathbf{a}_1\|^2\|\mathbf{a}_2\|^2 - (\mathbf{a}_1^T\mathbf{a}_2)^2$

(b) Area of $P = |\det(A)|$



16. If **x** and **y** are linearly independent vectors in $\mathbb{R}^3$, then they can be used to form a parallelogram P in the plane through the origin corresponding to Span(**x**, **y**). Show that

$$\text{Area of } P = \|\mathbf{x} \times \mathbf{y}\|$$

17. Let

$$\mathbf{x} = \begin{bmatrix} 4 \\ 4 \\ -4 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

(a) Determine the angle between **x** and **y**.

(b) Determine the distance between **x** and **y**.

18. Let **x** and **y** be vectors in $\mathbb{R}^n$ and define

$$\mathbf{p} = \frac{\mathbf{x}^T\mathbf{y}}{\mathbf{y}^T\mathbf{y}}\mathbf{y} \quad \text{and} \quad \mathbf{z} = \mathbf{x} - \mathbf{p}$$

(a) Show that $\mathbf{p} \perp \mathbf{z}$. Thus, **p** is the *vector projection* of **x** onto **y**; that is, $\mathbf{x} = \mathbf{p} + \mathbf{z}$, where **p** and **z** are orthogonal components of **x**, and **p** is a scalar multiple of **y**.

(b) If $\|\mathbf{p}\| = 6$ and $\|\mathbf{z}\| = 8$, determine the value of $\|\mathbf{x}\|$.

19. Use the database matrix U from Application 1 and search for the key words *orthogonality, spaces, vector*, only this time give the key word *orthogonality* twice the weight of the other two key words. Which of the eight modules best matches the search criteria? [*Hint*: Form the search vector using the weights 2, 1, 1 in the rows corresponding to the search words and then scale the vector to make it a unit vector.]

20. Five students in an elementary school take aptitude tests in English, mathematics, and science. Their scores are given in the table that follows. Determine the correlation matrix and describe how the three sets of scores are correlated.

	Scores		
Student	**English**	**Mathematics**	**Science**
S1	61	53	53
S2	63	73	78
S3	78	61	82
S4	65	84	96
S5	63	59	71
Average	66	66	76

21. Let t be a fixed real number and let

$$c = \cos t, \quad s = \sin t,$$
$$\mathbf{x} = (c, cs, cs^2, \ldots, cs^{n-1}, s^n)^T$$

Show that **x** is a unit vector in $\mathbb{R}^{n+1}$.

Hint:

$$1 + s^2 + s^4 + \cdots + s^{2n-2} = \frac{1 - s^{2n}}{1 - s^2}$$