6. $\mathbf{F} = (x^2y + x)\,\mathbf{i} + (y^3 - xy^2)\,\mathbf{j}$, D is the region inside the circle $x^2 + y^2 = 9$ and outside the circle $x^2 + y^2 = 4$.

7. (a) Use Green's theorem to calculate the line integral

$$\oint_C y^2 dx + x^2 dy,$$

where C is the path formed by the square with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$, oriented counterclockwise.

(b) Verify your answer for part (a) by calculating the line integral directly.

8. Let $\mathbf{F} = 3xy\,\mathbf{i} + 2x^2\,\mathbf{j}$ and suppose C is the oriented curve shown in Figure 28. Evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{s}$$

both directly and also by means of Green's theorem.

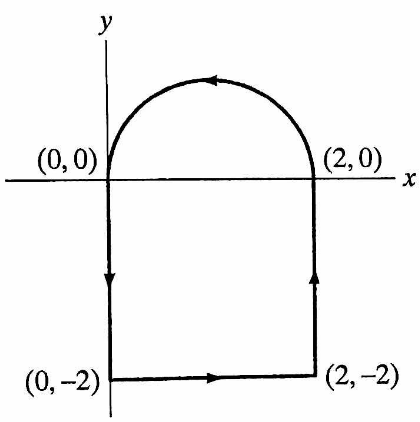

Figure 28 The oriented curve C of Exercise 8 consists of three sides of a square plus a semicircular arc.

9. Evaluate

$$\oint_C (x^2 - y^2)\,dx + (x^2 + y^2)\,dy,$$

where C is the boundary of the square with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$, oriented *clockwise*. Use whatever method of evaluation seems appropriate.

10. Use Green's theorem to find the work done by the vector field

$$\mathbf{F} = (4y - 3x)\,\mathbf{i} + (x - 4y)\,\mathbf{j}$$

on a particle as the particle moves counterclockwise once around the ellipse $x^2 + 4y^2 = 4$.

11. Verify that the area of the rectangle $R = [0, a] \times [0, b]$ is ab, by calculating an appropriate line integral.

12. Let a be a positive constant. Use Green's theorem to calculate the area under one arch of the cycloid

$$x = a(t - \sin t), \qquad y = a(1 - \cos t).$$

13. Evaluate $\oint_C (x^4y^5 - 2y)\,dx + (3x + x^5y^4)\,dy$, where C is the oriented curve pictured in Figure 29.

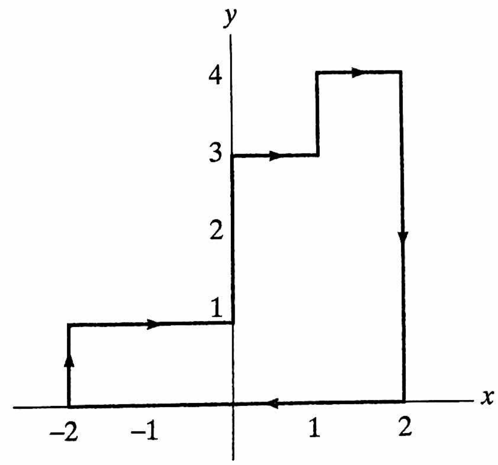

Figure 29 The oriented curve C of Exercise 13.

14. Use Green's theorem to find the area enclosed by the hypocycloid

$$\mathbf{x}(t) = (a\cos^3 t, a\sin^3 t), \quad 0 \le t \le 2\pi.$$

15. (a) Sketch the curve given parametrically by $\mathbf{x}(t) = (1 - t^2, t^3 - t)$.

(b) Find the area inside the closed loop of the curve.

16. Use Green's theorem to find the area between the ellipse $x^2/9 + y^2/4 = 1$ and the circle $x^2 + y^2 = 25$.

17. Show that if D is a region to which Green's theorem applies, and ∂D is oriented so that D is always on the left as we travel along ∂D, then the area of D is given by either of the following two line integrals:

$$\text{Area of } D = \oint_{\partial D} x\,dy = -\oint_{\partial D} y\,dx.$$

18. Find the area inside the quadrilateral whose vertices taken counterclockwise are $(2, 0)$, $(1, 2)$, $(-1, 1)$, and $(1, 1)$.

19. Suppose that the successive vertices of an n-sided polygon are the points (a_1, b_1), (a_2, b_2), … (a_n, b_n), arranged counterclockwise around the polygon. Show that the area inside the polygon is given by

$$\frac{1}{2}\left(\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} + \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} + \cdots + \begin{vmatrix} a_{n-1} & b_{n-1} \\ a_n & b_n \end{vmatrix} + \begin{vmatrix} a_n & b_n \\ a_1 & b_1 \end{vmatrix}\right).$$

20. Let a be a positive integer throughout this problem. An epicycloid is the path produced by a marked point on a circle of unit radius that rolls, without slipping,